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# Numerical Analysis of Lassa Fever Epidemic Model

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**ABSTRACT**

Lassa fever is an acute Hemorrhagic viral fever which is first discover in a town Lassa. In this paper we constructed a mathematical model to derive a relation of ordinary differential equations with saturated incident rate. Mathematical modelling is very useful tool in the field of epidemiology to study the behavior of diseases like COVID-19 Hepatitis B virus and Lassa fever etc. By using mathematical modeling, we analyze the existence and stability of the DFE and EE and find the Reproductive number R_0. The disease-free equilibrium is locally stable if R_0<1, and it is unstable if R_0>1 and disease endemic points are stable if R_0>1. The transmission dynamics of Lassa fever is analyzed numerically. In the present work two numerical schemes are developed which are standard finite difference (SFD) and non-standard finite difference scheme (NSFD). SFD scheme give conditionally convergence and do not behave well for certain parameter h. Our main purposed is to developed Non-Standard Finite Difference (NSFD) scheme which is unconditionally convergent for the Lassa fever model. Furthermore, we discuss the stability analysis of NSFD scheme. Finally, numerical experiments with all three schemes are presented to investigate the theoretically results.

## 1. Introduction

Lassa fever disease is spread in the contemporary of northern Nigeria and west Africa in 1969. It was first discovered when two female nurses are infected by this fever who works at Lassa mission hospital [1,2]. It spread from dead mice, and the duration of Lassa fever disease is 2 to 21 days. Lassa fever disease can be transfer from infected animal to human and also transfer from person to person. In northern Nigeria and West African kingdoms, about 2 to 3 million peoples are affected and death rate are 5000 to 10,000 individuals yearly [3-5]. Many researchers have participated and invested a

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significant amount of effort to investigating the dynamics of Lassa fever. In early 2018, over 300 confirmed positive cases of Lassa fever are reported in the month of March in Nigeria [6-9].

Mathematical modelling is a technique used to study the mechanisms that cause epidemic spread. It is also used to predict the possible fate of an infection and to evaluate epidemics control efforts [10]. Daniel Bernoulli, a trained physician, published the first explanation of mathematical modelling of disease spread in 1766. Bernoulli developed a mathematical model to secure the practice of immunization against Smallpox [11]. According to the calculations from this Framework, widespread vaccinations against Smallpox would improve life expectancy from two to three years of life. Different theoretical learnings have been designed on mathematical modelling of Lassa fever transmission dynamics concentrating on a number of various problems. In Okuonghae and Okuonghae [12] for the transmission of Lass fever illnesses, the author developed a SIS model paired with a rat population. They proposed a primary reproductive number for their framework as well as requirements for disease outbreaks. Mathematical Framework is a tool used to study the procedure by which disease breaks out. It's also used for analyzing the future direction of an outbreak and analyzing plans to control an epidemic [13-18].

The present paper is organized by the following manner. In section 1.1 flow chart of Lassa fever disease is constructed, and using this flow chart differential equations are derived for the mentioned disease. DFE and EE points of Lassa fever disease of model (1) are discussed in section 1.2. The most important threshold quantity which is known as basic reproduction number $R_0$ is find out in section 1.3. By using the reproduction number, in section 1.4 we find the stability of DFE and EE points. This stability analyses shows that the DFE points exist only when basic reproductive number is less than one and endemic points exit only when reproductive number is greater than one. In section 1.5, we constructed the numerical scheme such as Euler, Runga-Kutta of order _4 and NSFD scheme. In subsection of 1.5, we find the stability analyses for both disease free and endemic equilibrium points of NSFD scheme and also discussed that NSFD is unconditionally convergent at every step size and Euler and Runga-Kutta scheme are conditionally convergent. The comparison of numerical schemes are also discuss in the last subsection of 1.5. At the end of this article a brief conclusion is given.

### *1.1 Flow Chart*

The flow chart of model $(S, L, I, Is, R)$ is given below [19]

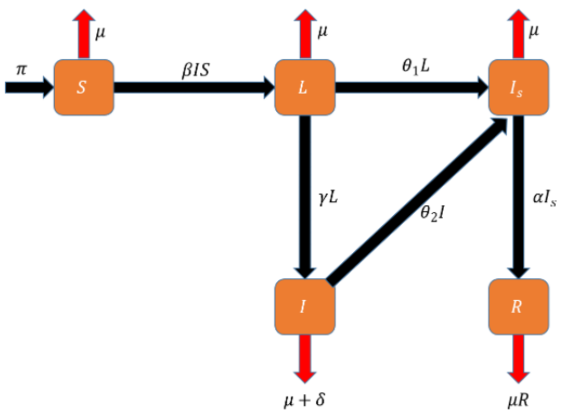


**Fig. 1.** Flow chart of model $(S, L, I, Is, R)$

We can explain the following nonlinear ordinary differential system using the flow chart above. The Lassa fever disease model can be used to develop differential equations such as:

$$\left.\begin{aligned} S' &= \frac{dS}{dt} = \pi - \beta IS - \mu S \\ L' &= \frac{dL}{dt} = \beta IS - \gamma L - \mu L - \theta_1 L \\ I' &= \frac{dI}{dt} = \gamma L - \mu I - \delta I - \theta_2 I \\ I_s' &= \frac{dI_s}{dt} = \theta_1 L + \theta_2 I - \mu I_s - \alpha I_s \\ R' &= \frac{dR}{dt} = \alpha I_s - \mu R \end{aligned}\right\}, \tag{1}$$

Where, $S(0) \geq 0, L(0) \geq 0, I(0) \geq 0, I_s(0) \geq 0, R(0) \geq 0$ and $S(t) + L(t) + I(t) + I_s(t) + R(t) \leq N$.

**Model Properties:** The feasible region $C = \{(S, L, I, I_s, R) \in R_+^5 : S(t) + L(t) + I(t) + I_s(t) + R(t) \leq N \, ; \, S(0) \geq 0, L(0) \geq 0, I(0) \geq 0, I_s(0) \geq 0, R(0) \geq 0\}$ at any time $t \geq 0$ and the solution of the model remnants positive and bounded.

**Property for positivity:** Consider equations of expression (1),

$$\frac{dS}{dt}|_{S=0} = \pi \geq 0 \qquad , \qquad \frac{dL}{dt}|_{L=0} = \beta \geq 0$$

$$\frac{dI}{dt}|_{I=0} = \gamma \geq 0 \qquad , \qquad \frac{dL}{dt}|_{I_s=0} = \theta_1, \theta_2 \geq 0$$

$$\frac{dI}{dt}|_{R=0} = \alpha \geq 0$$

**Parameters:**

The parameters of Lassa fever disease model [19] are given below:
$\pi$ Indicates the recruitment rate,
$\beta$ Indicate the contact rate of susceptible,
$\mu$ Indicates the natural death rate of individuals,
$\gamma$ Indicates individuals' rate of progression to the infection class,
$\theta_1$ Indicates the ratio at which latently infected people are isolated as a result of tracing
$\theta_2$ Indicates the rate at which persons with infections are quarantined,
$\delta$ Indicates the disease-related death,
$\alpha$ Indicates the rate of survival of isolated persons,
$S$ Indicates the Susceptible class,
$L$ Indicates the rate of latently,
$I$ Indicates the infected class,
$l_1$ Indicates the isolated class,

$R$ Indicates the recovered class,

**Parameters Values:**

**Table 1**
Parametric values

| Sr No# | Parameters | Values | Reference |
|---|---|---|---|
| 1. | $\pi$ | 0.9 | [19] |
| 2. | $\alpha$ | 0.6 | [19] |
| 3. | $\delta$ | 0.3 | [19] |
| 4. | $\theta_1$ | 0.5 | [19] |
| 5. | $\theta_2$ | 0.6 | [19] |
| 6. | $\beta$ | $0.05(DFE)$ | [19] |
| 7. | $\beta$ | $0.5(EE)$ | [19] |
| 8. | $\mu$ | 0.2 | [19] |
| 9. | $\gamma$ | 0.9 | [19] |

*1.2 Disease-Free and Endemic Equilibrium Points*

The formulated Mathematical model of Lassa fever has the disease free equilibrium at

$E_0(S,\ L,\ I,\ I_s,\ R) = \left(\frac{\pi}{\mu}, 0,0,0,0\right)$, and endemic points are $E^*(S^*, L^*, I^*, I_s^*, R^*)$

$$S^* = \frac{(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)}{\beta\gamma},$$

$$L^* = \frac{\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)}{\beta\gamma(\gamma + \mu + \theta_1)},$$

$$I^* = \frac{\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)}{\beta(\mu + \delta + \theta_2)(\gamma + \mu + \theta_1)},$$

$$I_s^* = \frac{\theta_1\big(\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)\big) + \theta_2\big(\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)\big)}{\beta(\mu + \delta + \theta_2)(\gamma + \mu + \theta_1)(\mu + \alpha)},$$

$$R^* = \frac{\alpha\Big(\theta_1\big(\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)\big) + \theta_2\big(\beta\gamma\pi - \mu(\gamma + \mu + \theta_1)(\mu + \delta + \theta_2)\big)\Big)}{\beta\gamma\mu(\mu + \delta + \theta_2)(\gamma + \mu + \theta_1)(\mu + \alpha)},$$

*1.3 Basic Reproductive Number* $(R_0)$

The most crucial threshold in every infectious disease is basic reproduction$R_0$. It can help predict if an infectious disease will spread through a population. Because our focus is on the population that spreads the infection, system (1) were considered, also with basic reproduction number obtained using the Next Generation Matrix [19].

$$F = \begin{pmatrix} 0 & 0 & -\beta\left(\frac{\pi}{\mu}\right) \\ 0 & 0 & \beta\left(\frac{\pi}{\mu}\right) \\ 0 & 0 & 0 \end{pmatrix},$$

$$\vartheta = \begin{pmatrix} -\mu & 0 & 0 \\ 0 & -\gamma-\mu-\theta_1 & 0 \\ 0 & \gamma & -\mu-\delta-\theta_2 \end{pmatrix},$$

Because we find $F\vartheta^{-1}$, so first of all we have to find the value of $\vartheta^{-1}$
As we know that,

$$\vartheta^{-1} = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \frac{1}{(\gamma+\mu+\theta_1)} & 0 \\ 0 & \frac{\gamma}{(\gamma+\mu+\theta_1)(\mu+\delta+\theta_2)} & \frac{1}{(\mu+\delta+\theta_2)} \end{bmatrix},$$

now we find $F\vartheta^{-1}$,

$$F\vartheta^{-1} = \begin{pmatrix} 0 & \frac{-\pi\beta\gamma}{\mu(\gamma+\mu+\theta_1)(\mu+\delta+\theta_2)} & \frac{-\pi\beta}{\mu(\mu+\delta+\theta_2)} \\ 0 & \frac{\pi\beta\gamma}{\mu(\gamma+\mu+\theta_1)(\mu+\delta+\theta_2)} & \frac{\pi\beta}{\mu(\mu+\delta+\theta_2)} \\ 0 & 0 & 0 \end{pmatrix},$$

thus,

$$R_0 = \frac{\pi\beta\gamma}{\mu(\gamma+\mu+\theta_1)(\mu+\delta+\theta_2)},$$

*1.4 Stability Analysis of Equilibria*

We assume that,

$$\left.\begin{array}{c} g_1 = \pi - \beta IS - \mu S \\ g_2 = \beta IS - \gamma L - \mu L - \theta_1 L \\ g_3 = \gamma L - \mu I - \delta I - \theta_2 I \\ g_4 = \theta_1 L + \theta_2 I - \mu I_s - \alpha I_s \\ g_5 = \alpha I_s - \mu R \end{array}\right\}, \tag{1}$$

**Theorem 1:** When$R_0 < 1$, so the DFE points are LAS (Locally Asymptotically Stable) for system (1).

**Proof:**

$$J=\begin{pmatrix}\frac{\partial g_1}{\partial S} & \frac{\partial g_1}{\partial L} & \frac{\partial g_1}{\partial I} & \frac{\partial g_1}{\partial I_s} & \frac{\partial g_1}{\partial R}\\ \frac{\partial g_2}{\partial S} & \frac{\partial g_2}{\partial L} & \frac{\partial g_2}{\partial I} & \frac{\partial g_2}{\partial I_s} & \frac{\partial g_2}{\partial R}\\ \frac{\partial g_3}{\partial S} & \frac{\partial g_3}{\partial L} & \frac{\partial g_3}{\partial I} & \frac{\partial g_3}{\partial I_s} & \frac{\partial g_3}{\partial R}\\ \frac{\partial g_4}{\partial S} & \frac{\partial g_4}{\partial L} & \frac{\partial g_4}{\partial I} & \frac{\partial g_4}{\partial I_s} & \frac{\partial g_4}{\partial R}\\ \frac{\partial g_5}{\partial S} & \frac{\partial g_5}{\partial L} & \frac{\partial g_5}{\partial I} & \frac{\partial g_5}{\partial I_s} & \frac{\partial g_5}{\partial R}\end{pmatrix},$$

$$J=\begin{pmatrix}\beta I-\mu & 0 & -\beta S & 0 & 0\\ \beta I & -(\gamma+\mu+\theta_1) & \beta S & 0 & 0\\ 0 & \gamma & -(\mu+\delta+\theta_2) & 0 & 0\\ 0 & \theta_1 & \theta_2 & -(\mu+\alpha) & 0\\ 0 & 0 & 0 & \alpha & -\mu\end{pmatrix},$$

$$J\left(\frac{\pi}{\mu},0,0,0,0\right)=\begin{pmatrix}-\mu & 0 & -\frac{\beta\pi}{\mu} & 0 & 0\\ \beta I & -(\gamma+\mu+\theta_1) & \frac{\beta\pi}{\mu} & 0 & 0\\ 0 & \gamma & -(\mu+\delta+\theta_2) & 0 & 0\\ 0 & \theta_1 & \theta_2 & -(\mu+\alpha) & 0\\ 0 & 0 & 0 & \alpha & -\mu\end{pmatrix},$$

$$|JE_0-\lambda I|=\begin{vmatrix}-\mu-\lambda & 0 & -\frac{\beta\pi}{\mu} & 0 & 0\\ \beta I & -(\gamma+\mu+\theta_1)-\lambda & \frac{\beta\pi}{\mu} & 0 & 0\\ 0 & \gamma & -(\mu+\delta+\theta_2)-\lambda & 0 & 0\\ 0 & \theta_1 & \theta_2 & -(\mu+\alpha)-\lambda & 0\\ 0 & 0 & 0 & \alpha & -\mu-\lambda\end{vmatrix},$$

$$\lambda_1=-\mu,$$

$$\lambda_2=-\mu,$$

$$\lambda_3=-(\mu+\alpha),$$

$$A=\begin{vmatrix}-(\gamma+\mu+\theta_1)-\lambda & \frac{\beta\pi}{\mu}\\ \gamma & -(\mu+\delta+\theta_2)-\lambda\end{vmatrix}.$$

Suppose,

$$P_1 = -(\gamma + \mu + \theta_1),$$

$$P_2 = \frac{\beta\pi}{\mu},$$

$$P_3 = \gamma,$$

$$P_4 = -(\mu + \delta + \theta_2).$$

So, above matrix $A$ becomes,

$$A = \begin{vmatrix} P_1 - \lambda & P_2 \\ P_3 & P_4 - \lambda \end{vmatrix} = 0,$$

$$(P_1 - \lambda)(P_4 - \lambda) - P_2P_3 = 0,$$

$$P_1P_4 - \lambda(P_1 + P_4) + \lambda^2 - P_2P_3 = 0,$$

$$\lambda^2 - \lambda(P_1 + P_4) + P_1P_4 - P_2P_3 = 0,$$

$$\lambda^2 - \lambda(P_1 + P_4) - (P_2P_3 - P_1P_4)(1 - R_0) > 0,$$

Whenever$R_0 < 1$, utilizing the Routh-Hurwitz criterion [20],21] the remaining roots of $\lambda^2 - \lambda(P_1 + P_4) - (P_2P_3 - P_1P_4) = 0$ must contain real negative aspects. Therefore, we deduce that $E_0$ is LAS for $R_0 < 1$.

**Theorem: 2** If $R_0 > 1$, then DEE point $E^*$ of model (1) is LAS.

Proof:

$$J(S^*, L^*, I^*, I_s^*, R^*) = \begin{pmatrix} \beta I^* - \mu & 0 & -\beta S^* & 0 & 0 \\ \beta I^* & -(\gamma + \mu + \theta_1) & \beta S^* & 0 & 0 \\ 0 & \gamma & -(\mu + \delta + \theta_2) & 0 & 0 \\ 0 & \theta_1 & \theta_2 & -(\mu + \alpha) & 0 \\ 0 & 0 & 0 & \alpha & -\mu \end{pmatrix},$$

$$|JE^* - \lambda I| = \begin{vmatrix} \beta I^* - \mu & 0 & -\beta S^* & 0 & 0 \\ \beta I^* & -(\gamma + \mu + \theta_1) & \beta S^* & 0 & 0 \\ 0 & \gamma & -(\mu + \delta + \theta_2) & 0 & 0 \\ 0 & \theta_1 & \theta_2 & -(\mu + \alpha) & 0 \\ 0 & 0 & 0 & \alpha & -\mu \end{vmatrix} = 0,$$

$$|JE^{*}-\lambda I|=\begin{vmatrix}(\beta I^{*}-\mu)-\lambda & 0 & -\beta S^{*} & 0 & 0\\ \beta I^{*} & -(\gamma+\mu+\theta_1)-\lambda & \beta S^{*} & 0 & 0\\ 0 & \gamma & -(\mu+\delta+\theta_2)-\lambda & 0 & 0\\ 0 & \theta_1 & \theta_2 & -(\mu+\alpha)-\lambda & 0\\ 0 & 0 & 0 & \alpha & -\mu-\lambda\end{vmatrix}=0,$$

$$\lambda_1=-\mu,$$

$$\lambda_2=-(\mu+\alpha),$$

$$|JE^{*}-\lambda I|=\begin{vmatrix}(\beta I^{*}-\mu)-\lambda & 0 & -\beta S^{*}\\ \beta I^{*} & -(\gamma+\mu+\theta_1)-\lambda & \beta S^{*}\\ 0 & \gamma & -(\mu+\delta+\theta_2)-\lambda\end{vmatrix}=0,$$

by putting

$$c_1=(\beta I^{*}-\mu),$$

$$c_2=-\beta S^{*},$$

$$c_3=\beta I^{*},$$

$$c_4=-(\gamma+\mu+\theta_1)-\lambda,$$

$$c_5=\beta S^{*},$$

$$c_6=\gamma,$$

$$c_7=-(\mu+\delta+\theta_2),$$

Now matrix $|JE^{*}-\lambda I|$ becomes.

$$|JE^{*}-\lambda I|=\begin{vmatrix}c_1-\lambda & 0 & c_2\\ c_3 & c_4-\lambda & c_5\\ 0 & c_6 & c_7-\lambda\end{vmatrix}=0,$$

$$(c_1-\lambda)\begin{vmatrix}c_4-\lambda & c_5\\ c_6 & c_7-\lambda\end{vmatrix}+c_2\begin{vmatrix}c_3 & c_4-\lambda\\ 0 & c_6\end{vmatrix}=0,$$

$$(c_1-\lambda)[(c_4-\lambda)(c_7-\lambda)-c_5c_6]+c_2(c_3c_6)=0,$$

$$(c_1-\lambda)[c_4c_7-c_4\lambda-c_7\lambda+\lambda^2]+c_2c_3c_6=0,$$

where,

$$C = c_2 c_3 c_6,$$

$$(c_1 - \lambda)[c_4 c_7 - \lambda(c_4 + c_7) + \lambda^2] + C = 0,$$

$$c_1 c_4 c_7 - c_1 \lambda(c_4 + c_7) + c_1 \lambda^2 - c_4 c_7 \lambda + \lambda^2(c_4 + c_7) - \lambda^3 + C = 0,$$

where,

$$G = c_1 c_4 c_7,$$

$$-\lambda^3 - \lambda^2(c_1 + c_4 + c_7) + \lambda(c_1 c_4 + c_1 c_7 - c_4 c_7) - G - C = 0, \quad \text{(A)}$$

$$-\lambda^3 - \lambda^2(c_1 + c_4 + c_7) + \lambda(c_1 c_4 + c_1 c_7 - c_4 c_7) - G - C(R_0 - 1) > 0,$$

By, applying Routh-Hurwitz criterion [20,21] all the roots of equation (A) must have negative real parts if and only $R_0 > 1$.Therefore, the DEE point $E^*$ is LAS.

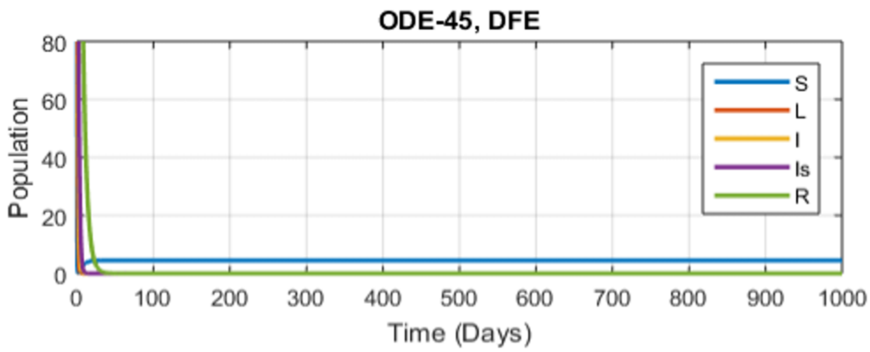


**Fig. 2.** Numerical Simulations of ODE-45 at point DFE

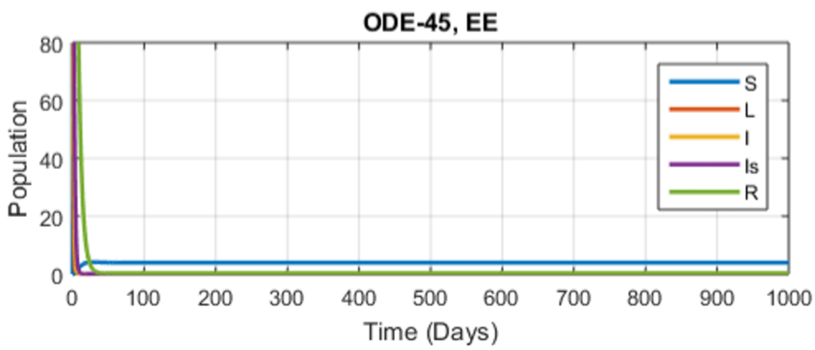


**Fig. 3.** Numerical Simulations of ODE-45 at point EE

Figure 2 shows the numerical results of Lassa fever disease of system (1) using ODE-45. In this Fig 1.2 graph shows the stability of DFE points and Figure 3 graph shows the stability of EE. The simulations result of above figures shows the stability of DFE at $\beta = 0.05$ and it shows the stability of endemic points at $\beta = 0.5$ remaining values of parameters are given in table 1.

*1.5 Numerical Analysis of Lassa Fever*

In this division we created three different schemes for system (1) of Lassa fever disease. In subsection 1.5.1 and 1.5.2, we worked on forward Euler and RK-4 schemes which shows convergence on small step size and if we increased step size then these schemes shows divergent results. In subsection 1.5.3 we constructed NSFD scheme which give unconditionally convergent and does not depend on step size.

*1.5.1 Forward Euler's Scheme:*

We developed Forward Euler scheme of Lassa fever of mathematical model for system (1).

$$\left.\begin{aligned} s^{n+1} &= s^n - h(\pi - \beta i^n s^n - \mu s^n) \\ l^{n+1} &= l^n + h(\beta i^n s^n - (\gamma - \mu - \theta_1) l^n) \\ i^{n+1} &= i^n + h[\gamma l^n - (\mu + \delta + \theta_2) i^n] \\ i_s^{n+1} &= i_s^n + h[\theta_1 l^n + \theta_2 i^n - (\mu + \alpha) i_s^n] \\ r^{n+1} &= r^n + h[\alpha i_s^n - \mu r^n] \end{aligned}\right\}, \tag{2}$$

After the solution of numerical work through Euler's structure give us the positivity results. When the step size rises, then the solution of Euler's structure does not remains stable. Thus, we conclude that the solution of Euler's structure is conditionally positive converge for all finite step size.

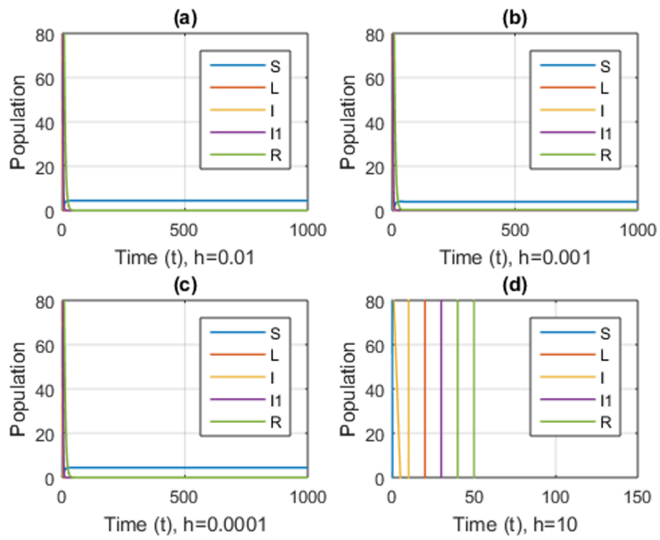


**Fig. 3.** Numerical Simulations of Euler's Scheme at point DFE

Numerical results in Fig 3 for Lassa fever disease are obtained from Euler schemes for DFE points which shows the conditionally convergence result at specific step size, as shown in graph (a) $h = 0.01$, (b) $h = 0.001$ and (c) $h = 0.0001$, and when we increase step size at (d) $h = 10$ it becomes diverged.

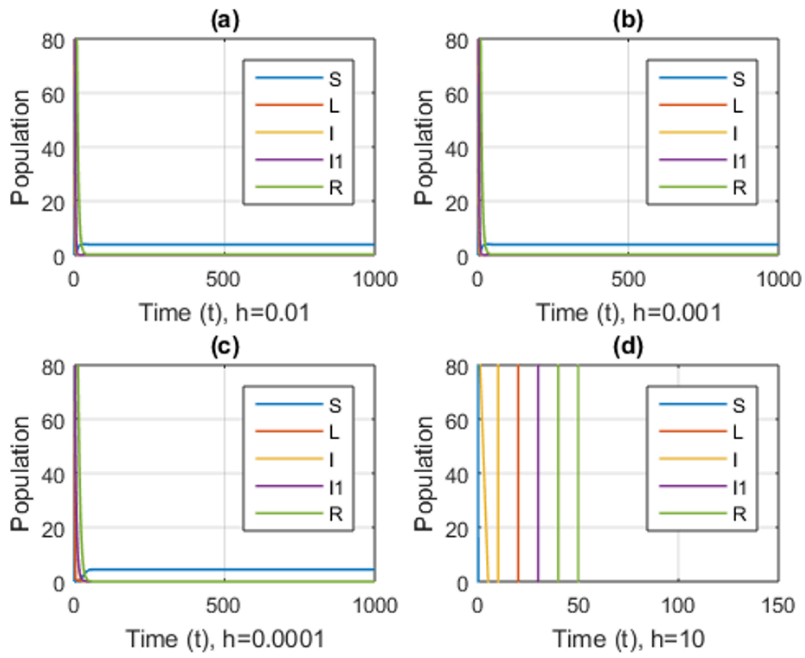


**Fig. 4.** Numerical Simulations of Euler's Scheme at point EE

In Fig 4 Euler scheme for endemic points of Lassa fever disease for system (1). The numerical results are given in Fig 4 graph (a), (b) and (c) shows the convergence results at $h = 0.01, 0.001$ and $h = 0.0001$ and (d) show the divergent of endemic points at $h = 10$.

*1.5.2 Fourth Order Runge-Kutta Scheme* $(RK-4)$*:*

We create $RK-4$ scheme for the mathematical modeling of Lassa fever for system (1),

$$\left.\begin{aligned} s^{n+1} &= s^n + \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4) \\ l^{n+1} &= l^n + \frac{1}{6}(t_1 + 2t_2 + 2t_3 + t_4) \\ i^{n+1} &= i^n + \frac{1}{6}(q_1 + 2q_2 + 2q_3 + q_4) \\ i_s^{n+1} &= i_s^n + \frac{1}{6}(v_1 + 2v_2 + 2v_3 + v_4) \\ r^{n+1} &= r^n + \frac{1}{6}(q_1 + 2q_2 + 2q_3 + q_4) \end{aligned}\right\}, \tag{3}$$

The numerical results obtained from this scheme are given below,

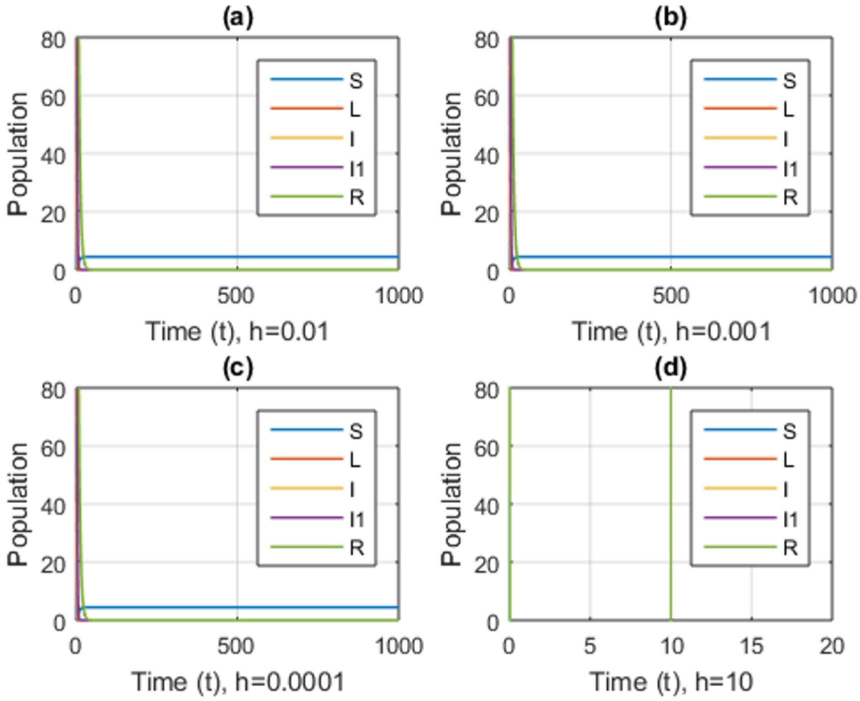


**Fig. 5.** Numerical Simulations of $RK-4$ Scheme at point DFE

The above Fig 5 the numerical simulations of system (1) through $RK-4$ scheme for DFE points, shows the conditionally convergent results at step size $h=0.01, 0.001$ and $0.0001$ respectively in (a), (b) and (c). As we increase the step size $h=10$ in (d) this scheme becomes diverge.

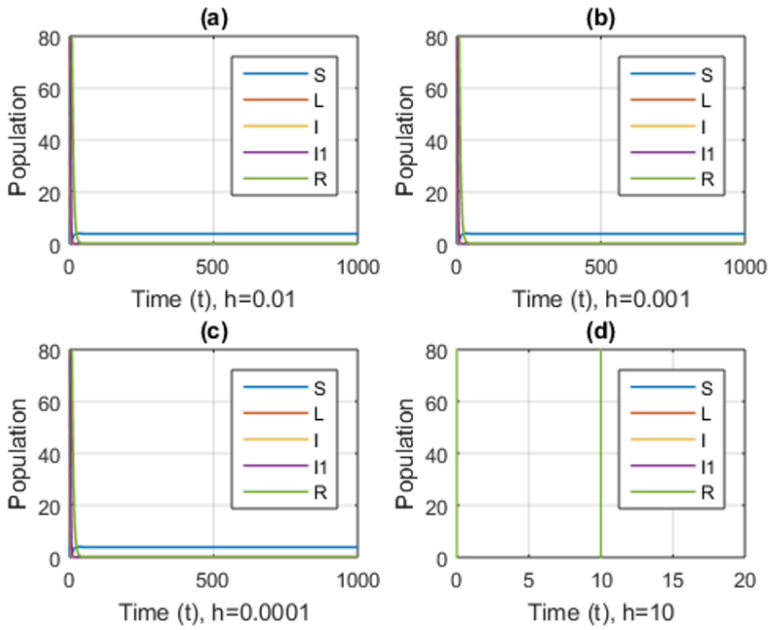


**Fig. 6.** Numerical Simulations of $RK-4$ Scheme at point EE

The above Fig 6 the numerical simulations of system (1) through $RK-4$ scheme for EE points, show the conditionally convergent results at step size $h = 0.01, 0.001$, and $0.0001$ respectively in (a), (b), and (c), as shown in above simulations results. As we increase values of $h = 10$ in (d) scheme becomes diverge.

### *1.5.3 Non-Standard Finite Difference (NSFD) Scheme*

In subsection 1.5.3 we constructed the most important unconditionally scheme for system (1), which is called non-standard finite difference scheme (NSFD). The NSFD method was first assembled by Mickens which is much better scheme from other two scheme like Euler and $RK-4$ scheme. For construction of NSFD scheme we use $(s^n, l^n, i^n, i_s^n, r^n)$ as numerical approximations of $S(t), l(t), i(t), r(t), i_s(t)$, at $t = nh$ and here $h$ is called step size of this scheme. The standard finite difference scheme are dependent on step size and it gives stability at some specific step size but the standard finite difference scheme are step size independent and show convergence at all finite step size at both DFE and EE points. So that we say that the NSFD scheme is most convenient scheme for the stability analysis of epidemic flow chart of Lassa fever virus [22-29].

$$\left.\begin{aligned} s^{n+1} &= \frac{s^n + h\pi}{(1 + h\beta i^n + h\mu)} \\ l^{n+1} &= \frac{l^n + h\beta i^n s^n}{(1 + h\gamma + h\mu + h\theta_1)} \\ i^{n+1} &= \frac{h\gamma l^n + i^n}{(1 + h\mu + h\delta + h\theta_2)} \\ i_s^{n+1} &= \frac{i_s^n + h\theta_1 l^n + h\theta_2 i^n}{(1 + h\mu + h\alpha)} \\ r^{n+1} &= \frac{r^n + h\alpha i_s^n}{(1 + h\mu)} \end{aligned}\right\}, \qquad (4)$$

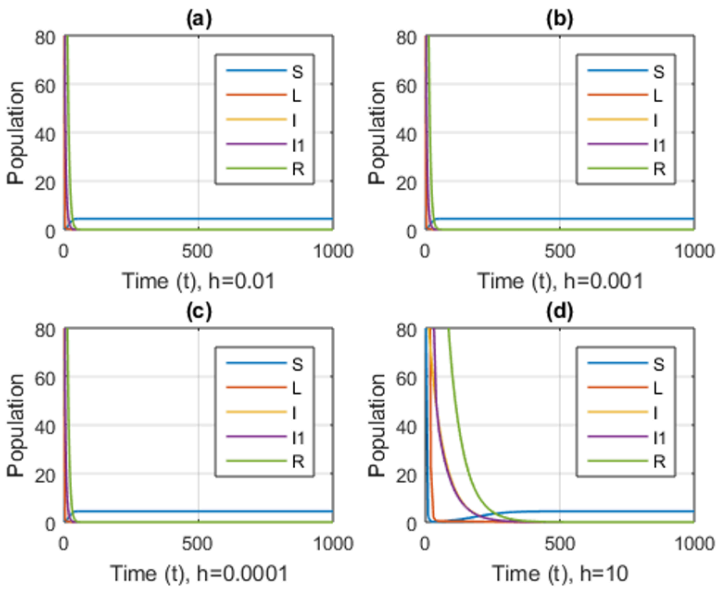


**Fig. 7.** Numerical Simulations of NSFD Scheme at point DFE

In Fig 7 the numerical results obtained through most valuable NSFD scheme for DFE points. The numerical simulations show unconditionally convergence as shown in Fig 7 (a), (b), (c) and (d). This simulations results conclude that NSFD scheme always show positive results at all finite step sizes.

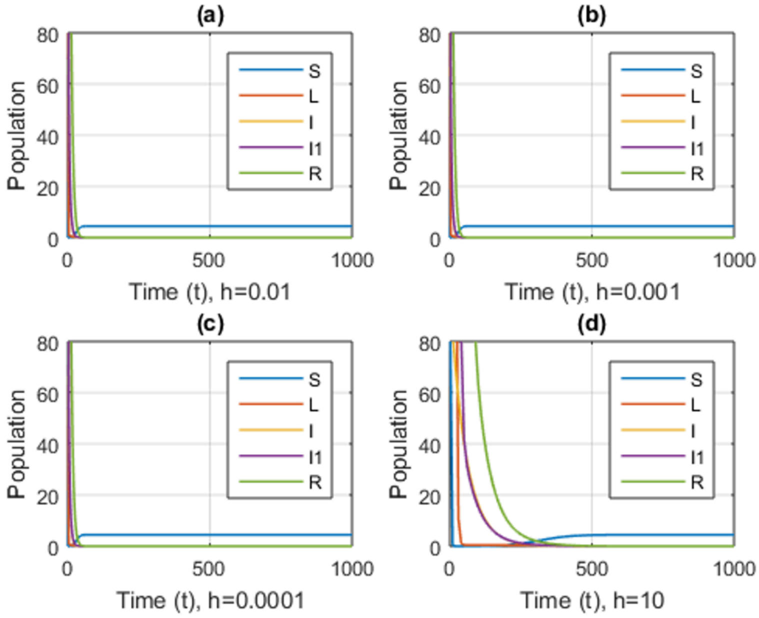


**Fig. 8.** Numerical Simulations of NSFD Scheme at point EE

In Fig 8 numerical simulations for system (1) shows the convergence results for endemic points through NSFD schemes. The graph (a), (b), (c) and (d) show the positive results for different step size which shows that the NSFD scheme is unconditionally convergent for endemic points.

*1.5.4 Stability analysis of NSFD scheme*

In this division we find the steadiness of NSFD method of system (1). Let us consider

$$\left.\begin{aligned} F = s^{n+1} &= \frac{s^n + h\pi}{(1 + h\beta i^n + h\mu)} \\ G = l^{n+1} &= \frac{l^n + h\beta i^n s^n}{(1 + h\gamma + h\mu + h\theta_1)} \\ H = i^{n+1} &= \frac{h\gamma l^n + i^n}{(1 + h\mu + h\delta + h\theta_2)} \\ I = i_s^{n+1} &= \frac{i_s^n + h\theta_1 l^n + h\theta_2 i^n}{(1 + h\mu + h\alpha)} \\ K = r^{n+1} &= \frac{r^n + h\alpha i_s^n}{(1 + h\mu)} \end{aligned}\right\}, \qquad (5)$$

**Theorem 3:**

If $R_0 < 1$ then DFE points of system (1) for NSFD scheme is LAS.

**Proof:**

Let's we take Jacobean matrix of order $5x5$,

$$J=\begin{pmatrix}\frac{\partial F}{\partial s} & \frac{\partial F}{\partial l} & \frac{\partial F}{\partial i} & \frac{\partial F}{\partial i_s} & \frac{\partial F}{\partial r}\\ \frac{\partial G}{\partial s} & \frac{\partial G}{\partial l} & \frac{\partial G}{\partial i} & \frac{\partial G}{\partial i_s} & \frac{\partial G}{\partial r}\\ \frac{\partial H}{\partial s} & \frac{\partial H}{\partial l} & \frac{\partial H}{\partial i} & \frac{\partial H}{\partial i_s} & \frac{\partial H}{\partial r}\\ \frac{\partial I}{\partial s} & \frac{\partial I}{\partial l} & \frac{\partial I}{\partial i} & \frac{\partial I}{\partial i_s} & \frac{\partial I}{\partial r}\\ \frac{\partial K}{\partial s} & \frac{\partial K}{\partial l} & \frac{\partial K}{\partial i} & \frac{\partial K}{\partial i_s} & \frac{\partial K}{\partial r}\end{pmatrix},$$

$$J=\begin{pmatrix}\frac{1}{1+h\beta i+h\mu} & 0 & 0 & 0 & 0\\ \frac{h\beta i}{1+h\gamma+h\mu+h\theta_1} & \frac{1}{1+h\gamma+h\mu+h\theta_1} & \frac{h\beta s}{1+h\gamma+h\mu+h\theta_1} & 0 & 0\\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2} & 0 & 0\\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)} & 0\\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu}\end{pmatrix},$$

$$J(E_0)=\begin{pmatrix}\frac{1}{1+h\mu} & 0 & 0 & 0 & 0\\ 0 & \frac{1}{1+h\gamma+h\mu+h\theta_1} & \frac{h\beta s}{1+h\gamma+h\mu+h\theta_1} & 0 & 0\\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2} & 0 & 0\\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)} & 0\\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu}\end{pmatrix},$$

$$J(E_0-\lambda)$$

$$=\begin{pmatrix} \frac{1}{1+h\mu}-\lambda & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1+h\gamma+h\mu+h\theta_1}-\lambda & \frac{h\beta s}{1+h\gamma+h\mu+h\theta_1} & 0 & 0 \\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2}-\lambda & 0 & 0 \\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)}-\lambda & 0 \\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu}-\lambda \end{pmatrix},$$

$$\lambda_1=\frac{1}{1+h\mu}>0,$$

$$\lambda_2=\frac{1}{1+h\mu}>0,$$

$$J(E_0-\lambda)=\begin{pmatrix} \frac{1}{1+h\gamma+h\mu+h\theta_1}-\lambda & \frac{h\beta s}{1+h\gamma+h\mu+h\theta_1} & 0 \\ \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2}-\lambda & 0 \\ \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)}-\lambda \end{pmatrix},$$

$$\lambda_3=\frac{1}{(1+h\mu+h\alpha)}>0,$$

Remaining eigen values are evaluated numerically and largest eigen values that is spectral radius graphs is plotted is given below.

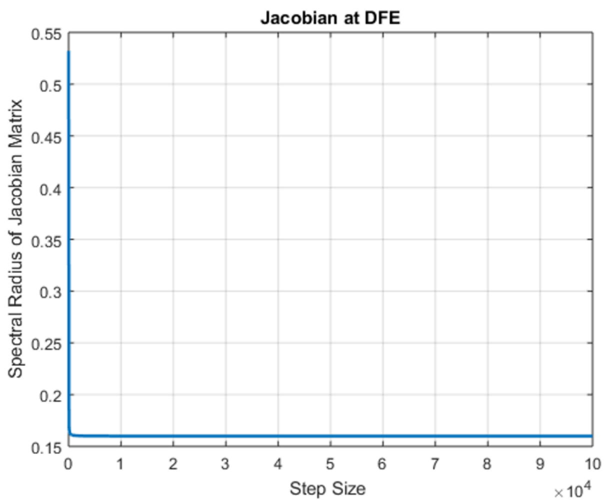


**Fig. 9.** Largest Eigen values of the system at point DFE

The figure 9 show that the largest eigen values is less than one at point DFE.

**Theorem: 4** If $R_0 > 1$, then DEE points $E^*$ of the discrete NSFD expression (1) is LAS $\forall\ h > 0$.

**Proof:** In the similar way as in Theorem 3, the Jacobean matrix can be obtained as

$$J = \begin{pmatrix} \frac{1}{1+h\beta i+h\mu} & 0 & 0 & 0 & 0 \\ \frac{h\beta i}{1+h\gamma+h\mu+h\theta_1} & \frac{1}{1+h\gamma+h\mu+h\theta_1} & \frac{h\beta s}{1+h\gamma+h\mu+h\theta_1} & 0 & 0 \\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2} & 0 & 0 \\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)} & 0 \\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu} \end{pmatrix},$$

By putting DEE point $E^*$, we get

$$J(E^*) = \begin{pmatrix} \frac{1}{1+h\beta i^*+h\mu} & 0 & 0 & 0 & 0 \\ \frac{h\beta i^*}{1+h\gamma+h\mu+h\theta_1} & \frac{1}{1+h\gamma+h\mu+h\theta_1} & \frac{h\beta s^*}{1+h\gamma+h\mu+h\theta_1} & 0 & 0 \\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2} & 0 & 0 \\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)} & 0 \\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu} \end{pmatrix},$$

$$|JE^* - \lambda I| = \begin{vmatrix} \frac{1}{1+h\beta i^*+h\mu} - \lambda & 0 & 0 & 0 & 0 \\ \frac{h\beta i^*}{1+h\gamma+h\mu+h\theta_1} & \frac{1}{1+h\gamma+h\mu+h\theta_1} - \lambda & \frac{h\beta s^*}{1+h\gamma+h\mu+h\theta_1} & 0 & 0 \\ 0 & \frac{h\gamma}{1+h\gamma+h\delta+h\theta_2} & \frac{1}{1+h\gamma+h\delta+h\theta_2} - \lambda & 0 & 0 \\ 0 & \frac{h\theta_1}{1+h\mu+h\alpha} & \frac{h\theta_2}{1+h\mu+h\alpha} & \frac{1}{(1+h\mu+h\alpha)} - \lambda & 0 \\ 0 & 0 & 0 & \frac{h\alpha}{1+h\mu} & \frac{1}{1+h\mu} - \lambda \end{vmatrix} = 0,$$

$$\lambda_1 = \frac{1}{1+h\mu} > 0,$$

$$\lambda_2 = \frac{1}{(1+h\mu+h\alpha)} > 0,$$

$$\lambda_3 = \frac{1}{1 + h\beta i^* + h\mu} > 0,$$

Remaining eigen values are evaluated numerically and largest eigen values that is spectral radius graphs is plotted is given below.

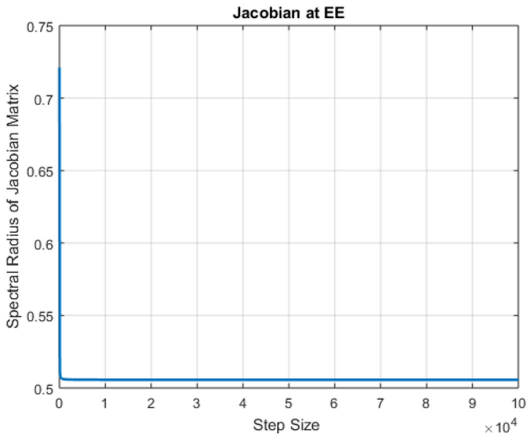


**Fig. 10.** Largest Eigen values of the system at point EE

The figure 10 show that the largest eigen values is less than one at point EE.

*1.5.5 Positivity*

The most significant physical characteristics of sub-population $(S, L, I, Is, R)$ used in the compartmental epidemic flow chart is positivity. The implicit numerical integration scheme and mathematical induction concept are used to study and ensure this fact.
**Theorem:5** Let the state variables $S(t), L(t), I(t), I_s(t)$ and $R(t)$ involved in the scheme are positive at $t = 0$; furthermore, if Additionally, every parameter is positive, then $S^{n+1} \geq 0, L^{n+1} \geq 0, I^{n+1} \geq 0, I_s^{n+1} \geq 0$ and $R^{n+1} \geq 0$.

**Proof:**
Using the mathematical induction principle and taking into consideration equation (4), we proceed on as follows:

$$S^{n+1} = \frac{S^n + h\pi}{(1 + h\beta I^n + h\mu)},$$

$$L^{n+1} = \frac{L^n + h\beta I^n S^n}{(1 + h\gamma + h\mu + h\theta_1)},$$

$$I^{n+1} = \frac{h\gamma L^n + I^n}{(1 + h\mu + h\delta + h\theta_2)},$$

$$I_s^{n+1} = \frac{I_s^n + h\theta_1 L^n + h\theta_2 I^n}{(1 + h\mu + h\alpha)},$$

$$R^{n+1} = \frac{R^n + h\alpha I_s^n}{(1 + h\mu)}$$

First, we Substitute $n = 0$ in above equations and reached.

$$S^1 = \frac{S^0 + h\pi}{(1 + h\beta I^0 + h\mu)} \geq 0 \qquad (6)$$

Similarly,

$$\left.\begin{aligned} L^1 &= \frac{L^0 + h\beta I^0 S^0}{(1 + h\gamma + h\mu + h\theta_1)} \geq 0 \\ I^1 &= \frac{h\gamma L^0 + I^0}{(1 + h\mu + h\delta + h\theta_2)} \geq 0 \\ I_s^1 &= \frac{I_s^0 + h\theta_1 L^0 + h\theta_2 I^0}{(1 + h\mu + h\alpha)} \geq 0 \\ R^1 &= \frac{R^0 + h\alpha I_s^0}{(1 + h\mu)} \geq 0 \end{aligned}\right\}, \qquad (8)$$

For $n = 1$, we arrive at.

$$S^2 = \frac{S^1 + h\pi}{(1 + h\beta I^1 + h\mu)} \geq 0 \qquad (9)$$

Similarly,

$$\left.\begin{aligned} L^2 &= \frac{L^1 + h\beta I^1 S^1}{(1 + h\gamma + h\mu + h\theta_1)} \geq 0 \\ I^2 &= \frac{h\gamma L^1 + I^1}{(1 + h\mu + h\delta + h\theta_2)} \geq 0 \\ I_s^2 &= \frac{I_s^1 + h\theta_1 L^1 + h\theta_2 I^1}{(1 + h\mu + h\alpha)} \geq 0 \\ R^2 &= \frac{R^1 + h\alpha I_s^1}{(1 + h\mu)} \geq 0 \end{aligned}\right\} \qquad (10)$$

Moreover, let us assume that the above system (10) ensures the positive property for the values of $n = 2,3,4, \dots, n-1$, i.e., $S^n \geq 0, L^n \geq 0, I^n \geq 0, I_s^n \geq 0$ and $R^n \geq 0$ for $n = 2,3,4, \dots, n-1$ and sub-population $(S, L, I, Is, R)$.
Now, for $n \in Z$,

$$S^{n+1} = \frac{S^n + h\pi}{(1 + h\beta I^n + h\mu)} \geq 0, \qquad (11)$$

Similarly,

$$\left.\begin{aligned} L^{n+1} &= \frac{L^n + h\beta I^n S^n}{(1 + h\gamma + h\mu + h\theta_1)} \geq 0 \\ I^{n+1} &= \frac{h\gamma L^n + I^n}{(1 + h\mu + h\delta + h\theta_2)} \geq 0 \\ I_s^{n+1} &= \frac{I_s^n + h\theta_1 L^n + h\theta_2 I^n}{(1 + h\mu + h\alpha)} \geq 0 \\ R^{n+1} &= \frac{R^n + h\alpha I_s^n}{(1 + h\mu)} \geq 0 \end{aligned}\right\}, \qquad (12)$$

Thus, proposed approach confirms the positive for the state variables $S(t), L(t), I(t), I_s(t)$ and $R(t)$ $\forall\, n \in Z^+$.

*1.5.6 Boundedness*

**Theorem:6** Suppose $S^0, L^0, I^0, I_s^0$ and $R^0$ are finite such that $S^0 + L^0 + I^0 + I_s^0 + R^0 \leq 1$. Also $\pi, \beta, \gamma, \theta_1, \theta_2$ and $\alpha$ are all conclusive in flow chart. Then discretized state variables, $S^{n+1}, L^{n+1}, I^{n+1}, I_s^{n+1}$ and $R^{n+1}$ are bounded by recrossing defined real constant $d_{n+1}$ such that $S^{n+1}, L^{n+1}, I^{n+1}, I_s^{n+1}$ and $R^{n+1} < d_{n+1}$ for all $n \in Z^+$ where $d_{n+1} = 4d_n + h\pi + I^n h(\beta S^n + \theta_2) + L^n h(\gamma + \theta_1) + h\alpha I_s^n$ and $d_{n+1} = 4 + h\pi + I^0 h(\beta S^0 + \theta_2) + L^0 h(\gamma + \theta_1) + h\alpha I_s^0$.
**Proof:** Examining the equations for the sub-population $(S, L, I, Is, R)$ in the implicit numerical integration method.

$$S^{n+1}(1 + h\beta I^n + h\mu) = S^n + h\pi \qquad (13)$$

$$L^{n+1}(1 + h\gamma + h\mu + h\theta_1) = L^n + h\beta I^n S^n \qquad (14)$$

$$I^{n+1}(1 + h\mu + h\delta + h\theta_2) = h\gamma L^n + I^n \qquad (15)$$

$$I_s^{n+1}(1 + h\mu + h\alpha) = I_s^n + h\theta_1 L^n + h\theta_2 I^n \qquad (16)$$

$$R^{n+1}(1 + h\mu) = R^n + h\alpha I_s^n \qquad (17)$$

By adding all above the equations.

$$\begin{aligned} \Rightarrow (S^{n+1} + L^{n+1} + I^{n+1} + I_s^{n+1} + R^{n+1})(1 + h\mu) + S^{n+1} h\beta I^n + L^{n+1} h(\gamma + \theta_1) + I^{n+1} h(\delta + \theta_2) \\ + I_s^{n+1} h\alpha \\ = (S^n + L^n + I^n + I_s^n + R^n) + h\pi + I^n h(\beta S^n + \theta_2) + L^n h(\gamma + \theta_1) \\ + h\alpha I_s^n \qquad (18) \end{aligned}$$

By substituting $n = 0$, in above equation (13).

$$\begin{aligned} \Rightarrow (S^1 + L^1 + I^1 + I_s^1 + R^1)(1 + h\mu) + S^1 h\beta I^0 + L^1 h(\gamma + \theta_1) + I^1 h(\delta + \theta_2) + I_s^1 h\alpha \\ = (S^0 + L^0 + I^0 + I_s^0 + R^0) + h\pi + I^0 h(\beta S^0 + \theta_2) + L^0 h(\gamma + \theta_1) + h\alpha I_s^0 \end{aligned}$$

$$\Rightarrow (S^1 + L^1 + I^1 + I_s^1 + R^1)(1 + h\mu) + S^1 h\beta I^0 + L^1 h(\gamma + \theta_1) + I^1 h(\delta + \theta_2) + I_s^1 h\alpha$$
$$< 5 + h\pi + I^0 h(\beta S^0 + \theta_2) + L^0 h(\gamma + \theta_1) + h\alpha I_s^0 = d_1$$

$$\Rightarrow (S^1 + L^1 + I^1 + I_s^1 + R^1)(1 + h\mu) + S^1 h\beta I^0 + L^1 h(\gamma + \theta_1) + I^1 h(\delta + \theta_2) + I_s^1 h\alpha \leq d_1$$

$$S^1(1 + h\mu + h\beta I^0) + L^1(1 + h\mu + h\gamma + h\theta_1) + I^1(1 + h\mu + h\delta + h\theta_2) + I_s^1(1 + h\mu + h\alpha)$$
$$+ R^1(1 + h\mu) \leq d_1$$

$$\Rightarrow S^1 < d_1$$

$$L^1 < d_1$$

$$I^1 < d_1$$

$$I_s^1 < d_1$$

$$R^1 < d_1$$

By substituting $n = 1$, in above equation (13).

$$\Rightarrow (S^2 + L^2 + I^2 + I_s^2 + R^2)(1 + h\mu) + S^2 h\beta I^1 + L^2 h(\gamma + \theta_1) + I^2 h(\delta + \theta_2) + I_s^2 h\alpha$$
$$= (S^1 + L^1 + I^1 + I_s^1 + R^1) + h\pi + I^1 h(\beta S^1 + \theta_2) + L^1 h(\gamma + \theta_1) + h\alpha I_s^1$$

$$\Rightarrow (S^2 + L^2 + I^2 + I_s^2 + R^2)(1 + h\mu) + S^2 h\beta I^1 + L^2 h(\gamma + \theta_1) + I^2 h(\delta + \theta_2) + I_s^2 h\alpha$$
$$< 5d_1 + h\pi + I^1 h(\beta S^1 + \theta_2) + L^1 h(\gamma + \theta_1) + h\alpha I_s^1 = d_1$$

$$\Rightarrow (S^2 + L^2 + I^2 + I_s^2 + R^2)(1 + h\mu) + S^2 h\beta I^1 + L^2 h(\gamma + \theta_1) + I^2 h(\delta + \theta_2) + I_s^2 h\alpha \leq d_1$$

$$\Rightarrow S^2(1 + h\mu + h\beta I^1) + L^2(1 + h\mu + h\gamma + h\theta_1) + I^2(1 + h\mu + h\delta + h\theta_2) + I_s^2(1 + h\mu + h\alpha)$$
$$+ R^2(1 + h\mu) \leq d_1$$

$$\Rightarrow S^2 < d_1$$

$$L^2 < d_1$$

$$I^2 < d_1$$

$$I_s^2 < d_1$$

$$R^2 < d_1$$

Now,

$$\Longrightarrow S^{n+1}(1+h\beta I^n+h\mu)+L^{n+1}(1+h\gamma+h\theta_1+h\mu)+I^{n+1}(1+h\mu+h\delta+h\theta_2)$$
$$+I_s^{n+1}(1+h\mu+h\alpha)+R^{n+1}(1+h\mu)$$
$$<5d_n+h\pi+I^n h(\beta S^n+\theta_2)+L^n h(\gamma+\theta_1)+h\alpha I_s^n=d_{n+1}$$

$S^{n+1}<d_{n+1}, L^{n+1}<d_{n+1}, I^{n+1}<d_{n+1}, {I_s}^{n+1}<d_{n+1}$ and $R^{n+1}<d_{n+1}$, where $d_{n+1}=4d_n+h\pi+I^n h(\beta S^n+\theta_2)+L^n h(\gamma+\theta_1)+h\alpha I_s^n=4+h\pi+I^0 h(\beta S^0+\theta_2)+L^0 h(\gamma+\theta_1)+h\alpha I_s^0$.

Hence, $S^{n+1}, L^{n+1}, I^{n+1}, {I_s}^{n+1}$ and $R^{n+1}$ are bordered by $\mathbb{R}$ $d_{n+1}$ $\forall\, n\in Z^+$.

### *1.5.7 Consistency analysis*

In subsection analyzes the consistency of an implicit numerical integration strategy using Taylor's series expansion. Firstly, choose expression (13) of the implicit numerical integration scheme and apply Taylor's series expansion of $S^{n+1}$.

$$S^{n+1}=S^n+h\frac{dS}{dt}+\frac{h^2}{2!}\frac{d^2S}{dt^2}+\frac{h^3}{3!}\frac{d^3S}{dt^3}+\cdots$$

In the following equation

$$S^{n+1}(1+h\beta I^n+h\mu)=S^n+h\pi$$

$$\left(S^n+h\frac{dS}{dt}+\frac{h^2}{2!}\frac{d^2S}{dt^2}+\frac{h^3}{3!}\frac{d^3S}{dt^3}+\cdots\right)(1+h\beta I^n+h\mu)=S^n+h\pi$$

$$S^n+S^n h\beta I^n+S^n h\mu+h\frac{dS}{dt}+h^2\beta I^n\frac{dS}{dt}+h^2\mu\frac{dS}{dt}+\left(\frac{h^2}{2!}\frac{d^2S}{dt^2}+\frac{h^3}{3!}\frac{d^3S}{dt^3}+\cdots\right)(1+h\beta I^n+h\mu)$$
$$=S^n+h\pi$$

$$S^n h\beta I^n+S^n h\mu+h\frac{dS}{dt}+h^2\beta I^n\frac{dS}{dt}+h^2\mu\frac{dS}{dt}+\left(\frac{h^2}{2!}\frac{d^2S}{dt^2}+\frac{h^3}{3!}\frac{d^3S}{dt^3}+\cdots\right)(1+h\beta I^n+h\mu)=h\pi$$

$$h\left(S^n\beta I^n+S^n\mu+\frac{dS}{dt}+h\beta I^n\frac{dS}{dt}+h\mu\frac{dS}{dt}+\left(\frac{h}{2!}\frac{d^2S}{dt^2}+\frac{h^2}{3!}\frac{d^3S}{dt^3}+\cdots\right)(1+h\beta I^n+h\mu)\right)=h\pi$$

$$S^n\beta I^n+S^n\mu+\frac{dS}{dt}+h\beta I^n\frac{dS}{dt}+h\mu\frac{dS}{dt}+\left(\frac{h}{2!}\frac{d^2S}{dt^2}+\frac{h^2}{3!}\frac{d^3S}{dt^3}+\cdots\right)(1+h\beta I^n+h\mu)=\pi$$

Apply $h\to 0$,

$$S^n\beta I^n+S^n\mu+\frac{dS}{dt}=\pi$$

$$\frac{dS}{dt} = \pi - S^n(\beta I^n + \mu) \tag{19}$$

Now, for equation (14) apply Taylor's series expansion.

$$L^{n+1} = L^n + h\frac{dL}{dt} + \frac{h^2}{2!}\frac{d^2L}{dt^2} + \frac{h^3}{3!}\frac{d^3L}{dt^3} + \cdots$$

$$L^{n+1}(1 + h\gamma + h\mu + h\theta_1) = L^n + h\beta I^n S^n$$

$$\left(L^n + h\frac{dL}{dt} + \frac{h^2}{2!}\frac{d^2L}{dt^2} + \frac{h^3}{3!}\frac{d^3}{dt^3} + \cdots\right)(1 + h\gamma + h\mu + h\theta_1) = L^n + h\beta I^n S^n$$

By applying $h \to 0$, we get the following expression.

$$\frac{dL}{dt} = \beta I^n S^n - L^n(\gamma + \mu + \theta_1) \tag{20}$$

We choose expression (15),

$$I^{n+1} = I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \cdots$$

$$I^{n+1}(1 + h\mu + h\delta + h\theta_2) = h\gamma L^n + I^n$$

$$\left(I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \cdots\right)(1 + h\mu + h\delta + h\theta_2) = h\gamma L^n + I^n$$

Apply $h \to 0$, and after some calculation we get.

$$\frac{dI}{dt} = \gamma L^n - I^n(\mu + \delta + \theta_2) \tag{21}$$

Pick the equation (16), then apply the Taylor's series expansion.

$$I_S^{n+1} = I_S^n + h\frac{dI_S}{dt} + \frac{h^2}{2!}\frac{d^2I_S}{dt^2} + \frac{h^3}{3!}\frac{d^3I_S}{dt^3} + \cdots$$

$$I_s^{n+1}(1 + h\mu + h\alpha) = I_s^n + h\theta_1 L^n + h\theta_2 I^n$$

$$\left(I_S^n + h\frac{dI_S}{dt} + \frac{h^2}{2!}\frac{d^2I_S}{dt^2} + \frac{h^3}{3!}\frac{d^3I_S}{dt^3} + \cdots\right)(1 + h\mu + h\alpha) = I_s^n + h\theta_1 L^n + h\theta_2 I^n$$

Put limit $h \to 0$, we get the result.

$$\frac{dI_S}{dt} = h\theta_1 L^n + h\theta_2 I^n - I_S^n(\mu + \alpha) \qquad (22)$$

From equation (17),

$$R^{n+1} = R^n + h\frac{dR}{dt} + \frac{h^2}{2!}\frac{d^2R}{dt^2} + \frac{h^3}{3!}\frac{d^3R}{dt^3} + \cdots$$

$$R^{n+1}(1 + h\mu) = R^n + h\alpha I_s^n$$

$$\left(R^n + h\frac{dR}{dt} + \frac{h^2}{2!}\frac{d^2R}{dt^2} + \frac{h^3}{3!}\frac{d^3R}{dt^3} + \cdots\right)(1 + h\mu) = R^n + h\alpha I_s^n$$

By applying $h \to 0$, we reached.

$$\frac{dR}{dt} = \alpha I_s^n - R^n h\mu \qquad (23)$$

*1.5.8 Comparison analysis of SFD and NSFD schemes*

The comparison analysis of both numerical schemes are presented in this section, which shows the reliability of these schemes.

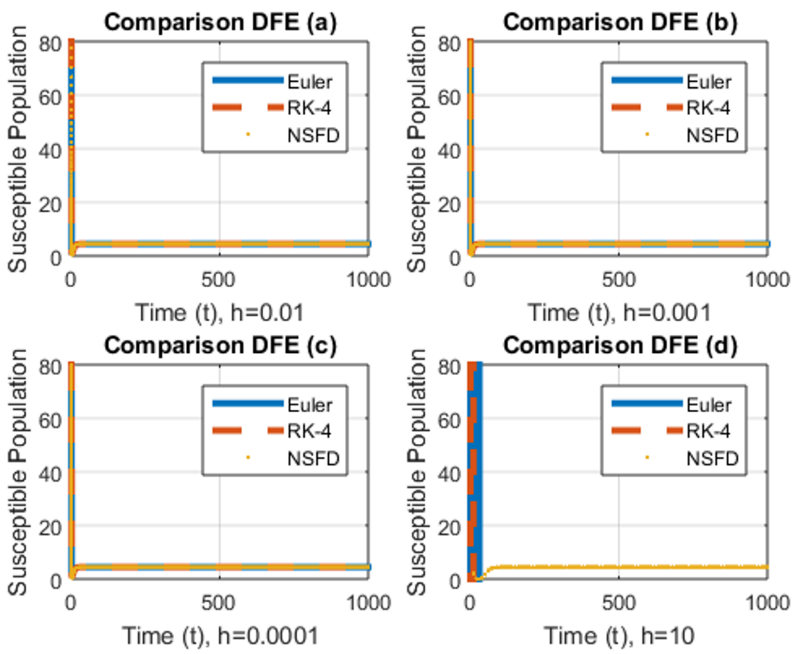


**Fig. 11.** Numerical Simulations of NSFD Scheme at point DFE

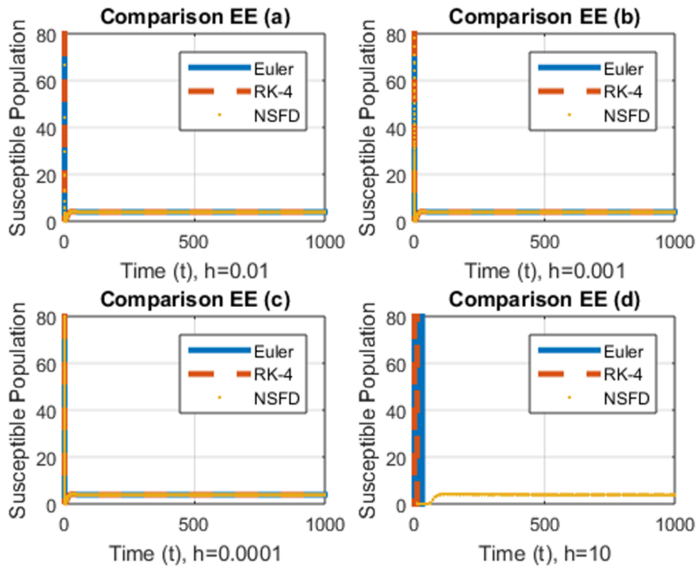


**Fig. 12.** Numerical simulations of NSFD Scheme at point EE

Figure 12 Numerical results represents the comparison of SFD and NSFD schemes. In Fig 1.9, graph (a) and (b) gives the simulations of DFE point and graph (c) and (d) give the simulations for EE point.

## 2. Conclusion

A mathematical model for Lassa fever disease are examined in this paper. A formulated mathematical model consist of five variables which are expressed in differential equations. We find disease free and endemic points by using these differential equations, basic reproductive value is used to find the stability of mathematical model of Lassa fever. Furthermore, we use two different numerical schemes such as SFD and NSFD scheme to numerically analyze the flow chart of Lassa fever virus. We see that in SFD scheme shows conditionally convergent for both endemic and disease free equilibrium points. After that we constructed the most frequent scheme for numerically analyze the Lassa fever disease model which is known as NSFD scheme. This scheme gives us unconditionally convergent results for both DFE and EE points. It means that it does not depend on step, it remain stable at all finite step size and give convergence result. Numerical simulations are formed for all above schemes to validate the theoretical and mathematical work.

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