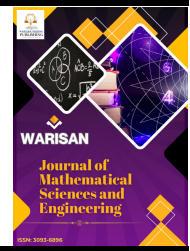




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Neutrosophic $\alpha\beta\gamma$ -Slice Based Reduction Framework for B-Spline Surface Modeling of Kenyir Lake Bathymetry Data

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ABSTRACT

Modeling bathymetric surfaces often involves managing the uncertainty inherent in collected data. Type-2 neutrosophic sets (T2NS) provide a robust mathematical framework to represent this uncertainty, especially when truth, indeterminacy, and falsity values vary over secondary domains. However, the practical interpretation and visualization of type-2 neutrosophic B-spline surface (T2NBsS) models remain computationally challenging due to their complex structure. This research addresses the problem of simplifying T2NBsS interpolation models for real-world applications, particularly in the context of Kenyir Lake bathymetry data. The main objective is to visualize a reduction technique that transforms T2NBsS into a type-1 neutrosophic B-spline surface (T1NBsS) form without significantly losing the accuracy of the information. To achieve this, we propose an α -slice based vertical interval approximation strategy that slices the secondary membership structure at selected α levels to extract interval values of the type-1 triangular neutrosophic set. This method bypasses iterative centroid computation and enables efficient surface reconstruction while preserving essential neutrosophic characteristics. The results confirm that the proposed method effectively bridges the gap between theoretical modeling and practical geospatial interpretation. This framework offers a valuable tool for researchers and engineers dealing with uncertain spatial datasets in environmental and hydrographic domains.

1. Introduction

Accurate modeling of bathymetric surfaces is crucial in environmental analysis, reservoir management, and hydrographic applications. Bathymetry data often contain uncertainty due to measurement errors, sparse sampling, and environmental fluctuations. These uncertainties pose challenges for traditional modeling techniques such as deterministic interpolation or type-1 fuzzy systems, which lack the ability to represent complex or layered uncertainty [1,2].

To overcome these limitations, type-2 fuzzy sets (T2FS) were introduced as an extension of type-1 fuzzy sets (T1FS), where the membership grades themselves are fuzzy [3]. This approach enables

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better handling of uncertainties in input information [4]. The practical use of T2FS was further developed by Mendel and Karnik, who proposed type-reduction algorithms such as the Karnik–Mendel (KM) algorithm [4]. In parallel, the concept of neutrosophic sets was introduced by Smarandache, allowing representation of truth, indeterminacy, and falsity independently [5]. Type-2 neutrosophic sets (T2NS), which combine the strengths of T2FS and neutrosophic logic, have since been proposed to model more complex forms of uncertainty [6].

In surface modeling, B-spline surfaces are widely used due to their continuity, flexibility, and local control over surface shape. When combined with T2NS theory, a type-2 neutrosophic B-spline surface (T2NBsS) offers a powerful tool to interpolate uncertain spatial data while preserving multi-dimensional uncertainty characteristics [9,10]. However, practical use of T2NBsS models is hindered by the computational burden of processing secondary membership structures and the difficulty in interpreting high-dimensional uncertainty in real applications [6,10].

To address this issue, researchers have proposed approximation strategies using α -cuts on the secondary membership functions. This method creates vertical slices that capture bounded uncertainty intervals without relying on iterative centroid computations [7,8]. These α -slice strategies provide an efficient and interpretable way to simplify general Type-2 fuzzy models [6].

This paper proposes an $\alpha\beta\gamma$ -slice based type-reduction framework for converting a T2NBsS into a type-1 neutrosophic B-spline surface (T1NBsS). The method simplifies the secondary membership structure into interval-valued representations based on selected α levels. A case study on Kenyir Lake bathymetry is conducted to demonstrate the effectiveness of this approach. The results show that the reduced T1NBsS closely approximates the original T2NBsS with improved computational efficiency and interpretability.

The remainder of this paper is organized as follows. Section 2 provides a review of related work on fuzzy and neutrosophic interpolation models and type-reduction techniques. Section 3 introduces the mathematical formulation of the type-2 neutrosophic B-spline surface interpolation model. Section 4 presents the proposed $\alpha\beta\gamma$ -slice based type-reduction method in detail. Section 5 discusses the case study involving Kenyir Lake bathymetry, including results and analysis. Finally, Section 6 concludes the study and outlines potential future work.

2. Related Works

The modeling of uncertainty in surface interpolation in this study has been deeply influenced by advances in fuzzy set theory and neutrosophic logic. Zadeh's foundational work on fuzzy sets [1] introduced the concept of gradual membership, where a T1FS $\tilde{A}$ in universe X is defined as:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \mid x \in X, \mu_{\tilde{A}}(x) \in [0,1] \right\} \quad (1)$$

where $\mu_{\tilde{A}}(x)$ is the membership function that quantifies the degree of belonging of element x to the set.

To handle higher levels of uncertainty, Zadeh extended this idea into T2FS [3], where the membership function itself is fuzzy. A T2FS $\tilde{\tilde{A}}$ is formally defined as:

$$\tilde{\tilde{A}} = \left\{ \left(x, \tilde{\mu}_{\tilde{\tilde{A}}}(x) \right) \mid x \in X \right\} \quad (2)$$

where:

$$\tilde{\mu}_{\tilde{\tilde{A}}}(x) = \left\{ \left(x, \mu_{\tilde{\mu}_{\tilde{\tilde{A}}}}(x, u) \right) \mid u \in [0,1], \mu_{\tilde{\mu}_{\tilde{\tilde{A}}}}(x, u) \in [0,1] \right\}$$

Here, each primary membership x has a secondary membership function over the domain $u \in [0,1]$ forming a three-dimensional membership space.

In parallel, Smarandache proposed neutrosophic sets (NS) [11], a powerful generalization of intuitionistic and fuzzy sets. A single-valued neutrosophic set $\hat{A}$ in universe X is defined by three independent membership functions:

$$\hat{A} = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \right\} \quad (3)$$

where:

- $T_A(x) \in [0,1]$: truth-membership degree
- $I_A(x) \in [0,1]$: indeterminacy-membership degree
- $F_A(x) \in [0,1]$: falsity-membership degree

Unlike intuitionistic fuzzy sets, neutrosophic sets allow $T + I + F$ to range freely within $[0,3]$, providing more expressiveness.

To further enhance the capacity for uncertainty representation, researchers proposed T2NS, where each of the three components becomes a fuzzy set [12]:

$$\hat{\hat{A}} = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \right\} \quad (4)$$

with:

$$\begin{aligned} \hat{\hat{T}}_A(x) &= \left\{ \langle t, \mu_{T_A(x)}(t) \rangle \mid t \in [0,1], \mu_{T_A(x)}(t) \in [0,1] \right\} \\ \hat{\hat{I}}_A(x) &= \left\{ \langle i, \eta_{I_A(x)}(i) \rangle \mid i \in [0,1], \eta_{I_A(x)}(i) \in [0,1] \right\} \\ \hat{\hat{F}}_A(x) &= \left\{ \langle f, \nu_{F_A(x)}(f) \rangle \mid f \in [0,1], \nu_{F_A(x)}(f) \in [0,1] \right\} \end{aligned}$$

This advanced framework supports multi-level uncertainty modeling for each dimension of information: truth, indeterminacy, and falsity to making it highly suitable for complex systems like bathymetry.

Despite these advancements, most practical work in neutrosophic geometric modeling, such as Bézier curves and B-spline surfaces, has focused on type-1 or interval-based representations [9,10,13-22]. Several works have introduced deneutrosophication strategies to convert neutrosophic representations to crisp values, and neutrosophication to fuzzified precise data into neutrosophic domains [23-25]. However, a gap remains: type-reduction of type-2 neutrosophic geometric models, particularly for B-spline surface interpolation, has not been systematically studied.

This paper fills this gap by proposing an $\alpha\beta\gamma$ -slice-based vertical decomposition strategy to reduce a T2NBsS into a T1NBsS. This method preserves uncertainty semantics while improving computational efficiency and interpretability for real-world modeling, such as lake bathymetry.

3. Type-2 Neutrosophic B-spline Interpolation Model

The type-1 neutrosophic B-spline interpolation model has been introduced in [9]. Therefore, a type-2 neutrosophic B-spline interpolation model will be introduced based on the study and the definition of T2NS features as follows:

Definition 1 (Type-2 neutrosophic B-spline surface interpolation)

Let $\hat{\hat{P}} = \left\{ \hat{\hat{P}}_{i,j} \right\}_{i=1, j=1}^{n+1, m+1}$ as a set of type-2 neutrosophic control net relations (T2NCNR). Thus, the tensor product type-2 neutrosophic B-spline surface is given as

$$\tilde{\tilde{B}}sS(u, w) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j} N_i^k(u) M_j^l(w) \quad (5)$$

with $N_i^k(u)$ and $M_j^l(w)$ are B-spline basis functions in the u and w directions, respectively, can be written as follows:

$$N_i^1(u) = \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$N_i^k(u) = \frac{(u - u_i)}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{(u_{i+k} - u)}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u)$$

$$M_j^1(w) = \begin{cases} 1 & \text{if } w_j \leq w < w_{j+1} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

$$M_j^l(w) = \frac{(w - w_j)}{w_{j+l-1} - w_j} M_j^{l-1}(w) + \frac{(w_{j+l} - w)}{w_{j+l} - w_{j+1}} M_{j+1}^{l-1}(w)$$

with u_i and v_j are the elements of the knot vectors.

Next is the type-2 neutrosophic B-spline surface for the truth T , indeterminacy I , and falsity F for primary memberships while truth μ , indeterminacy η , and falsity ν for secondary memberships can be described as in Definition 2. The definition will only fuzzified the z-axis through Eq. (5) as follows:

Definition 2 (Type-2 Neutrosophic B-spline Surface for T, I, F, μ, η and ν)

Let $\hat{\hat{P}}_{i,j}^{T,I,F}$ and $\hat{\hat{P}}_{i,j}^{\mu,\eta,\nu}$ as the T2NCNR for truth, indeterminacy, and falsity memberships for primary and secondary memberships, respectively. Thus, the definition of T2NBsS for truth, indeterminacy, and falsity's primary and secondary memberships are as follows:

$$\tilde{\tilde{B}}sS(u, w) = \left\langle \begin{matrix} BsS^T(u, w), BsS^I(u, w), BsS^F(u, w), \\ BsS^\mu(u, w), BsS^\eta(u, w), BsS^\nu(u, w) \end{matrix} \right\rangle \quad (8)$$

with

$$\sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j} N_i^k(u) M_j^l(w) = \left\langle \begin{matrix} \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^T N_i^k(u) M_j^l(w), \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^I N_i^k(u) M_j^l(w), \\ \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^F N_i^k(u) M_j^l(w), \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^\mu N_i^k(u) M_j^l(w), \\ \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^\eta N_i^k(u) M_j^l(w), \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} \hat{\hat{P}}_{i,j}^\nu N_i^k(u) M_j^l(w) \end{matrix} \right\rangle \quad (9)$$

if the T, I, F, μ, η and ν follows the conditions in Eq. (3).

The type-2 neutrosophic B-spline surface interpolation is an important method in geometric modeling because it can help in visualizing this study problem. The generation of T2NBsS interpolation is obtained by defining a control mesh for the surface that interpolates the data points. Therefore, for each known surface data point, Eq. (5) gives a linear equation to the vertices of the control mesh that are unknown, $\hat{P}_{i,j}$. Eq. (5) can be written as a single surface data point and produce a system of simultaneous equations in matrix form as follows:

$$\begin{bmatrix} \hat{D} \end{bmatrix} = [F] \begin{bmatrix} \hat{P} \end{bmatrix} \quad (10)$$

with $[F] = N_i^k M_j^l$, $\begin{bmatrix} \hat{D} \end{bmatrix}$ is T2NDN and $\begin{bmatrix} \hat{P} \end{bmatrix}$ is T2NCNR. Thus, the T2NCNR can be obtained as follows:

$$\begin{bmatrix} \hat{P} \end{bmatrix} = [F]^{-1} \begin{bmatrix} \hat{D} \end{bmatrix} \quad (11)$$

The parameter values for u_1 and w_1 for each surface data point are obtained using the approximation method length of the span. Specifically, for data point r , the parameter value at data point k^{th} in the parameter direction, u is

$$u_1 = 0$$

$$\frac{u_k}{u_{\max}} = \frac{\sum_{r=2}^k \left| \hat{D}_r - \hat{D}_{r-1} \right|}{\sum_{r=2}^j \left| \hat{D}_r - \hat{D}_{r-1} \right|}, \quad k \geq 2 \quad (12)$$

and for data point s , the parameter value at data point l^{th} in the parameter direction, w is

$$w_1 = 0$$

$$\frac{w_l}{w_{\max}} = \frac{\sum_{s=2}^l \left| \hat{D}_s - \hat{D}_{s-1} \right|}{\sum_{s=2}^j \left| \hat{D}_s - \hat{D}_{s-1} \right|}, \quad l \geq 2 \quad (13)$$

with $u_{\max}$ and $w_{\max}$ are the maximum values that correspond to the knot vector.

4. Neutrosophic Type-Reduction by using α -Slice Based Vertical Interval Approximation

Type-reduction is a key operation in type-2 fuzzy systems, serving as a bridge between a general type-2 fuzzy set (GT2FS) and a usable type-1 fuzzy output [3]. Traditional methods, such as the Karnik–Mendel (KM) algorithm, reduce interval type-2 Fuzzy sets (IT2FS) to interval-valued Type-1 sets by iteratively computing centroid bounds. In modeling uncertainty in GT2FS, the secondary membership function plays a critical role in quantifying the extent of imprecision at each primary domain value. One effective way to simplify and interpret this uncertainty is by applying α -cuts on the secondary membership functions, forming vertical slices that yield bounded uncertainty intervals. Liu [6] and Yeh et al. [7] proposed decomposition strategies using α -cuts to extract interval type-2 fuzzy slices from GT2FSs. These vertical slices represent bounded uncertainty at each primary value, providing a means for interval-based modeling and centroid approximation without the need for computationally intensive iterations.

The $Interval_i = [z_i - \mu_L^\alpha, z_i + \mu_U^\alpha]$ is constructed by performing α -level slicing on the secondary membership function of a Type-2 fuzzy set, where the left and right spreads are computed as

$\mu_L^\alpha = (c_i - pl_i) \cdot \alpha$ and $\mu_U^\alpha = (pr_i - c_i) \cdot \alpha$. These expressions define the α -scaled deviation of the primary value z from its left and right bounds PL and PU , respectively. Although Mendel and John [4] laid the foundational concepts using vertical slices and the footprint of uncertainty (FOU), this explicit interval construction is formalized and applied in later works on α -cut and α -plane representation of Type-2 fuzzy sets [5,6].

While type reduction comes next, after the fuzzification step has been completed. The defuzzification operation in type-1 Fuzzy Logic Sets (FLSs) is an "expanded" version of type-reduction in a type-2 FLS [3]. To enable the denutrosophication of the type-1 NS, neutrosophic type-reduction is defined and applied to T2NCNR first. After the neutrosophication, it will get a T2NCNR, $\hat{P}_i^{T,I,F,\mu,\eta,\nu}$. Then, do the neutrosophic type-reduction to reduce the T2NCNR, $\hat{P}_i^{T,I,F,\mu,\eta,\nu}$ to type-1 neutrosophic control net relation (T1NCNR), $\hat{P}_i^{T,I,F}$. This method extends the GT2FS α -slice-based vertical interval approximation proposed by [7], as described in Algorithm 1.

Algorithm 1: Neutrosophic Type-Reduction of T2NCNR

Input:

$\hat{P}_i^{T,I,F} = \{\hat{P}_1^{T,I,F}, \hat{P}_2^{T,I,F}, \dots, \hat{P}_n^{T,I,F}\}$: Primary (crisp) values

$P_{i(L)}^{\mu,\eta,\nu} = [P_1^{\mu,\eta,\nu}, P_2^{\mu,\eta,\nu}, \dots, P_n^{\mu,\eta,\nu}]$: Lower bounds of secondary membership

$P_{i(R)}^{\mu,\eta,\nu} = [P_1^{\mu,\eta,\nu}, P_2^{\mu,\eta,\nu}, \dots, P_n^{\mu,\eta,\nu}]$: Upper bounds of secondary membership

$\hat{\alpha}, \hat{\beta}, \hat{\gamma} \in [0,1]$: α, β, γ -level for slicing

Output: $Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}]$: $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ -slice interval around each

$\hat{P}_i^{T,I,F}$

Steps:

For each primary value $\hat{P}_i^{T,I,F} \in Z$:

Compute the center: $c_i = \frac{P_{i(L)}^{\mu,\eta,\nu} + P_{i(R)}^{\mu,\eta,\nu}}{2}$ and c_i is specific for the asymmetric case.

Compute the left α -cut offset: ${}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}} = (c_i - P_{i(L)}^{\mu,\eta,\nu}) \cdot \langle \hat{\alpha}, \hat{\beta}, \hat{\gamma} \rangle$

Compute the right α -cut offset: ${}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}} = (P_{i(R)}^{\mu,\eta,\nu} - c_i) \cdot \langle \hat{\alpha}, \hat{\beta}, \hat{\gamma} \rangle$

Compute the interval around $\hat{P}_i^{T,I,F}$: $Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}]$

Return all interval $Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}] \equiv [\hat{P}_i^{T,I,F(L)}, \hat{P}_i^{T,I,F(R)}]$ as the α, β, γ -slice vertical approximation.

Based on Algorithm 1 which is neutrosophic type-reduction of T2NCNR, this section will use the algorithm for T2NBsS. Thus, the algorithm is as follows:

Algorithm 2: Neutrosophic Type-Reduction of T2NBsS

Input:

$\hat{P}_i^{T,I,F} = \{\hat{P}_1^{T,I,F}, \hat{P}_2^{T,I,F}, \dots, \hat{P}_n^{T,I,F}\}$: Primary (crisp) values

$P_{i(L)}^{\mu,\eta,\nu} = [P_1^{\mu,\eta,\nu}, P_2^{\mu,\eta,\nu}, \dots, P_n^{\mu,\eta,\nu}]$: Lower bounds of secondary membership

$P_{i(R)}^{\mu,\eta,\nu} = [P_1^{\mu,\eta,\nu}, P_2^{\mu,\eta,\nu}, \dots, P_n^{\mu,\eta,\nu}]$: Upper bounds of secondary membership

$\hat{\alpha} = 0.2, \hat{\beta} = 0.8, \hat{\gamma} = 1 \in [0,1]$: $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ -level for slicing

Output: $Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}]$: $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ -slice interval around each

$\hat{P}_i^{T,I,F}$

Steps:

For each primary value $\hat{P}_i^{T,I,F} \in Z$:

Compute the center: $c_i = \frac{P_{i(L)}^{\mu,\eta,\nu} + P_{i(R)}^{\mu,\eta,\nu}}{2}$ and c_i is specific for the asymmetric case.

Compute the left α -cut offset: ${}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}} = (c_i - P_{i(L)}^{\mu,\eta,\nu}) \cdot \langle \hat{\alpha}, \hat{\beta}, \hat{\gamma} \rangle$

Compute the right α -cut offset: ${}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}} = (P_{i(R)}^{\mu,\eta,\nu} - c_i) \cdot \langle \hat{\alpha}, \hat{\beta}, \hat{\gamma} \rangle$

Compute the interval around $\hat{P}_i^{T,I,F}$: $Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}]$

Return all interval

$Interval_i = [\hat{P}_i^{T,I,F} - {}^{T,I,F}\mu_L^{\hat{\alpha},\hat{\beta},\hat{\gamma}}, \hat{P}_i^{T,I,F} + {}^{T,I,F}\mu_U^{\hat{\alpha},\hat{\beta},\hat{\gamma}}] \equiv [\hat{P}_i^{T,I,F(L)}, \hat{P}_i^{T,I,F(R)}] \equiv {}^{\alpha,\beta,\gamma}\bar{B}sS^{T,I,F}(t)$ as the

α, β, γ -slice vertical approximation.

5. Neutrosophic Type-Reduction by using Bathymetry Data at Kenyir Lake

The degree for each function is determined based on boat fluctuations, weather factors, traces recorded on graphics or digital records, as well as Zulkifly [26]'s experience and perception of the data. Based on this reason, the degrees of truth, falsity, and indeterminacy applied to the entire bathymetry data are 0.8, 0.1, and 0.3, respectively. Referring to Smarandache [27], the equal distribution method is used to produce bathymetry data collection degrees for membership and non-membership functions of 0.95 and 0.25, respectively. Therefore, the triangle of type-2 neutrosophic numbers is illustrated as in Fig. 1.

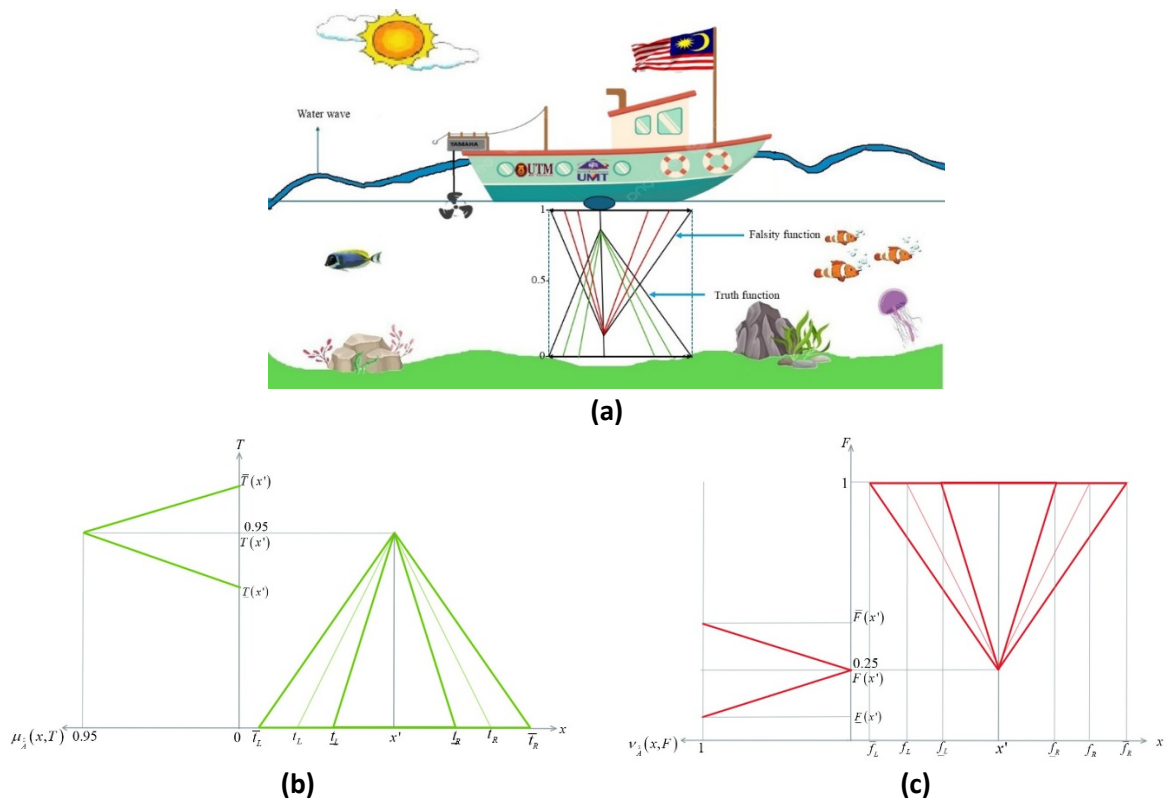
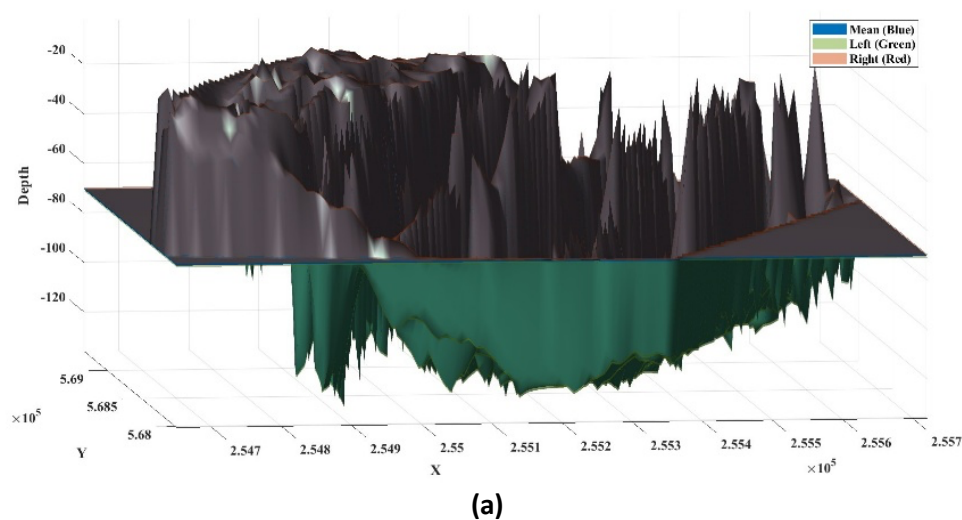
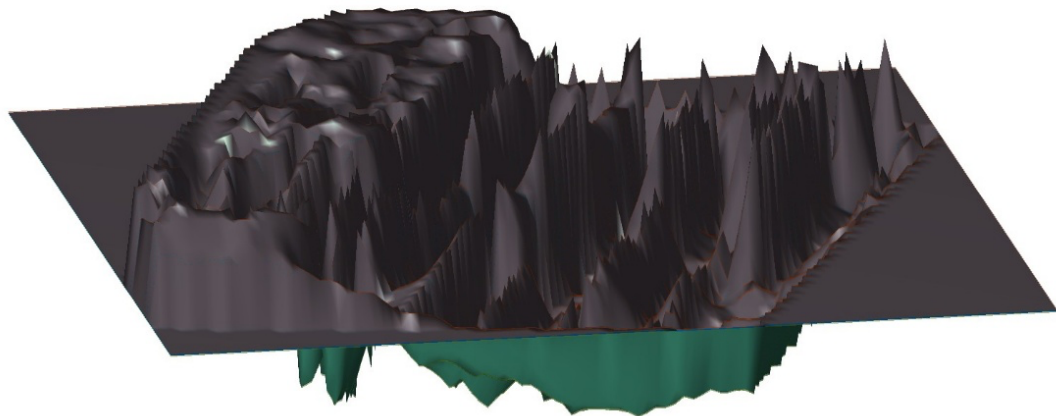


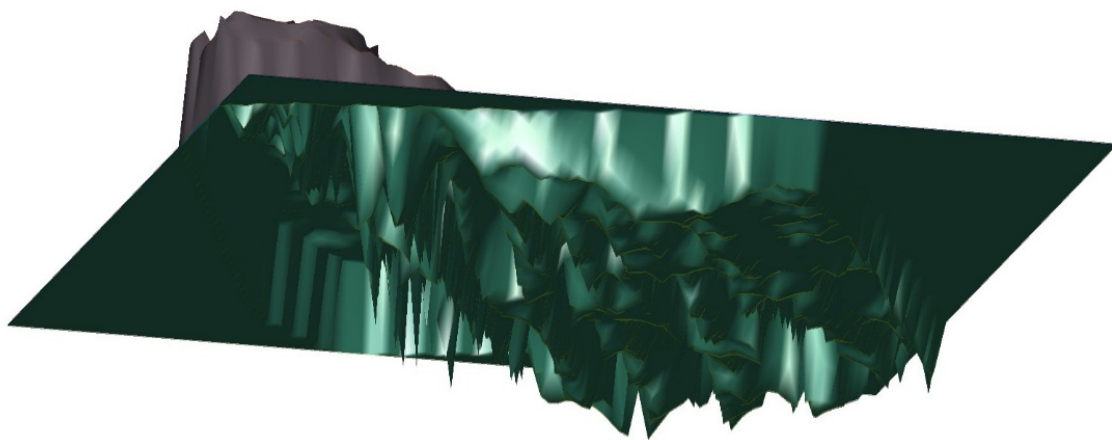
Fig. 1. Illustration of triangle T2NN for data in Zone 1; **(a)** Based on the real situation; **(b)** Triangle T2NN for truth membership; and **(c)** Triangle T2NN for falsity membership

The neutrosophic type-reduction model of T2NBsS interpolation will be demonstrated as it transforms into the type-1 neutrosophic B-spline surface (T1NBsS) model in this section by using Algorithm 3. Fig. (2) and (3) visualize the T1NBsS models for truth and falsity memberships, respectively, for bathymetry data in Zone 1, based on the highest and lowest values, which occur when $\alpha = 0.95$ and $\gamma = 0.25$.



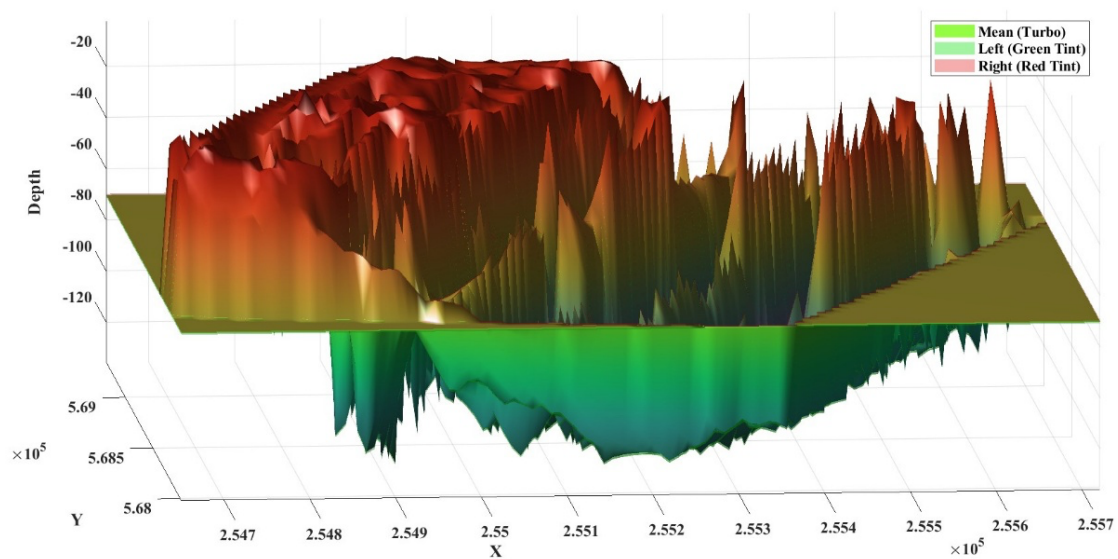


(b)

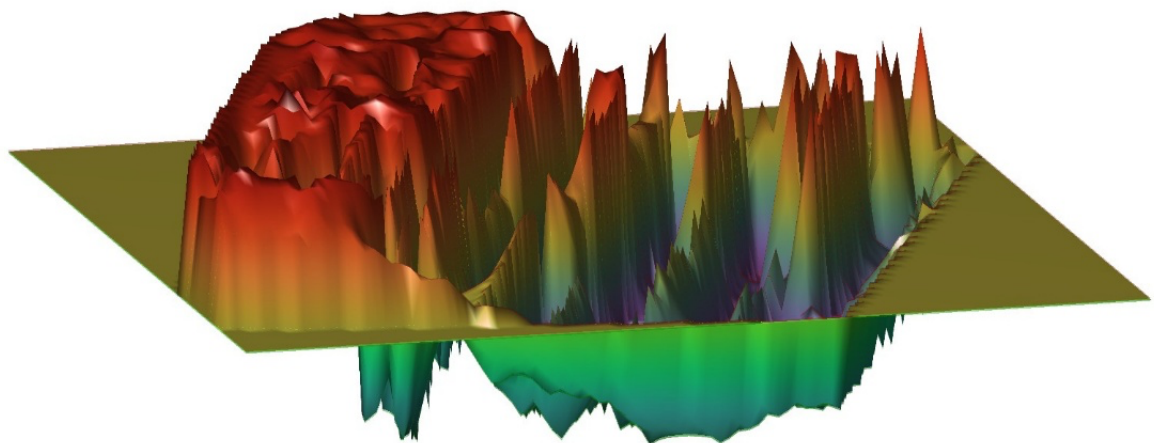


(c)

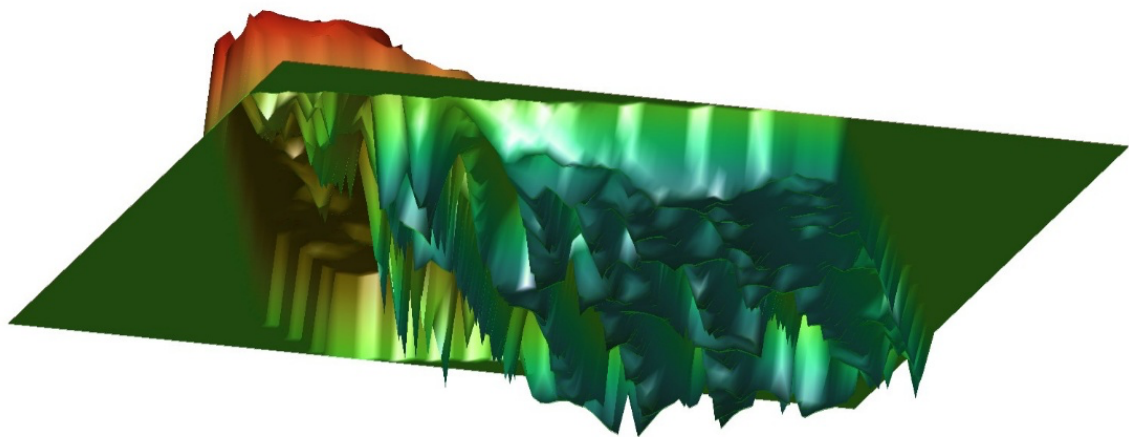
Fig. 2. Type-1 neutrosophic B-spline surface interpolation models for truth membership: (a) With axis, (b) Without axis, (c) Different view



(a)



(b)



(c)

Fig. 3. Type-1 neutrosophic B-spline surface interpolation models for falsity membership: (a) With axis, (b) Without axis, (c) Different view

The results illustrated in Fig. 2 and Fig. 3 depict the reduced T1NBsS for truth and falsity memberships of Kenyir Lake Zone 1. These surfaces were derived from the original type-2 neutrosophic model through $\alpha\gamma$ -slice-based type-reduction. As supported by Liu [6] and Yeh et al. [7], when the α -level increases, the left and right bounds of the uncertainty interval contract toward the center, reflecting increased confidence. Similarly, at lower γ -levels representing less dominance of falsity, the left and right bounds also approach the central mean, indicating reduced ambiguity. This consistent convergence behavior ensures a more precise neutrosophic interval representation under high α or low γ scenarios.

In Fig. 2, which presents the truth membership surface, the green-colored regions indicate the deepest parts of Kenyir Lake, where the truth membership is highest under the selected α -level. These are zones where the model has high certainty that the bathymetric values correspond to deep areas. In contrast, the grey regions represent shallower parts of the lake, where the truth value is lower, reflecting reduced confidence in depth. Meanwhile, in Fig. 3, which shows the falsity membership surface, the color pattern conveys the opposite. The red regions represent shallow areas where falsity membership is high, indicating a strong rejection of depth, while the green areas once again correspond to deeper zones, where the falsity value is low. These complementary surfaces help distinguish bathymetric depth and shallowness while incorporating uncertainty into the

interpretation. The topography revealed through this dual-component neutrosophic surface allows for accurate localization of zones with varying bathymetric certainty, emphasizing the practicality of component-wise analysis for complex terrain like lakebeds.

The neutrosophic type-reduction process introduced here offers key advantages. First, it simplifies the multidimensional structure of type-2 neutrosophic sets into manageable interval representations without iterative computation, unlike the Karnik–Mendel method [3]. Second, it improves the clarity of interpretation by maintaining the separate contributions of truth, indeterminacy, and falsity within the model. This allows users to better understand how each type of uncertainty affects the data. Finally, by enabling practical visualization of uncertainty across geographical surfaces, the approach becomes especially valuable for environmental analysis and hydrographic applications, where decisions often depend on both the accuracy of the data and how confidently that data can be trusted.

5. Conclusions

This study introduced a novel $\alpha\beta\gamma$ -slice-based type-reduction framework for transforming a T2NBsS into a T1NBsS. The proposed method simplifies the complex secondary membership structures of truth, indeterminacy, and falsity components by applying vertical decomposition at selected $\alpha\beta\gamma$ -levels. This non-iterative, component-wise strategy effectively preserves uncertainty characteristics while significantly improving computational tractability and interpretability.

Application of the method to Kenyir Lake Zone 1 bathymetry demonstrated its ability to reveal spatial uncertainty patterns in lakebed depth modeling. Analysis of the resulting surfaces showed that increasing α -levels and decreasing γ -values lead to convergence of interval bounds toward the mean. Moreover, the type-reduced surfaces clearly distinguished between deep and shallow regions, validating the model's ability to support detailed terrain interpretation.

Overall, the proposed framework presents theoretical advancements in neutrosophic set theory and fuzzy geometric modeling with real-world geospatial applications. Future research may consider extending this method by incorporating additional environmental factors relevant to bathymetric modeling, such as salinity distribution, sediment transport, or turbidity, which can influence depth readings and uncertainty interpretation. From a modeling perspective, more advanced geometric techniques like non-uniform rational B-splines (NURBS) could be explored to improve shape flexibility and continuity in complex lakebed topographies. Furthermore, the current neutrosophic framework could be compared or combined with other uncertainty-based theories such as Pythagorean fuzzy sets, q-rung Orthopair fuzzy sets, or intuitionistic fuzzy logic, to assess their relative effectiveness in handling multi-dimensional imprecision in spatial and environmental datasets.

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