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An Efficient Approach to Approximate Analytical Solutions of Second-Order Nonlinear Telegraph Equations

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ABSTRACT

We propose the Multistep Modified Reduced Differential Transform Method (MMRDTM) as a novel approach to solving Nonlinear Telegraph Equations (NLTEs). To streamline the process, we replace the nonlinear terms within NLTEs with corresponding Adomian polynomials before implementing the multistep technique. This substitution simplifies the solution process and enables more precise approximations over longer time domains. To validate the MMRDTM's efficacy and accuracy, we solve two different NLTE problems, showcasing the method's capability for analytical approximation. The resulting outcomes are then presented both in tabular and graphical formats. The findings confirm that MMRDTM delivers highly accurate, and in some cases, exact solutions for the studied equations.

1. Introduction

The one-dimensional Nonlinear Telegraph Equations (NLTE) of form in Al-Badrani *et al.,* [1] have been taken into consideration

$$w_{tt} - w_{xx} + aw_t + \Phi(w) = h(x,t), \tag{1}$$

the initial conditions are given as follows

$$w(x,0) = h_1(x),$$

$$w_t(x,0) = h_2(x),$$

with $a > 0$ is a constant, Φ is a function of w while h is a function of x and t. Eq. (1) emerges in the investigation of the transmission of electrical signals within transmission line cables and in the

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analysis of wave phenomena. A variety of nonlinear phenomena occurring within physical and biological processes can be attributed to the interaction between convection and diffusion. This interaction is also known as the mutual action of reaction and diffusion. In addition, the telegraph equation is better suited for modelling reaction-diffusion in such fields of research than the ordinary diffusion equation, as stated by Mittal and Bhatia [2].

Various numerical and analytical techniques can be deployed to acquire solution of the telegraph equation. For instance, Mohebbi and Dehghan [3] investigated high-order compact solutions to obtain solution of the telegraph equation. Then, Gao and Chi [4] proposed an unconditionally stable difference scheme for a 1D linear hyperbolic equation. Saadatmandi and Dehghan [5] formulated a numerical solution utilizing the Chebyshev Tau method. Furthermore, Yousefi [6] applied the Legendre multi-wavelet Galerkin method to obtain the solution of the hyperbolic telegraph equation. A numerical method for dealing with the second-order two-space-dimensional telegraph equation based on truly meshless local weak-strong (MLWS) methods was devised by Dehghan and Ghesmati [7]. In [8, 9], Adomian decomposition method is deployed to compute the solution of the telegraph equation. Recently, Sayed *et al.,* [10] solved nonlinear telegraph equations by applying the Adomian Decomposition Method with an accelerated formula of Adomian polynomial.

Many powerful and effective methods for approximating analytical solutions have been iteratively improved such as the Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM), Variation Iteration Method (VIM), Hirota's Bilinear Method, Balance Method, Inverse Scattering Method, and Differential Transform Method (DTM). The PDEs with complicated nonlinearity are hard to solve by the existing method due to their nonlinearity term. The existing methods also diverge in wide time area.

Therefore, in [11], Ray modified the fractional RDTM for the fractional KdV equation. This method has been modified by replacing the nonlinear term with related Adomian polynomials. Consequently, the nonlinear problem's solutions can be obtained more simply and with fewer computed terms. Later, El-Zahar [12] developed an adaptive multi-step DTM for solving singular perturbation initial-value problems by producing a solution in the form of a quickly converging sequence that achieves convergence over an extended period of time. These two methods are proposed in this paper for solving second-order nonlinear Telegraph equations.

The Multistep Modified Reduced Differential Transform Method (MMRDTM) was introduced by Che Hussin *et al.,* [13] for the purpose of obtaining solutions to Nonlinear Schrödinger Equations (NLSEs). Furthermore, in [14], Che Hussin *et al.,* evaluated the efficacy of the MMRDTM in approximating solutions to the Klein-Gordon equations. Subsequently, Che Hussin *et al.,* [15] utilized the MMRDTM to determine solutions for fractional NLSEs. Che Hussin *et al.,* [16] also utilized the method for solving the nonlinear KdV equation. Solutions for NLSEs with power-law nonlinearity were recently obtained by Che Hussin *et al.,* [17] using MMRDTM. The approximation results are obtained through a reduction in the number of calculated terms, while maintaining high precision. In addition, results converge rapidly over an extended time frame.

The multistep approach and the modification by adopting Adomian polynomials are embedded in this paper to perform the MMRDTM for solving nonlinear telegraph equations (NLTEs). Furthermore, we employed parametrization methods to generate Adomian polynomials without requiring time-consuming high-derivative calculations as proposed in Kataria and Vellaisamy [18]. In addition, Sabdin *et al.,* [19] proposed a novel method named the Adaptive Hybrid Reduced Differential Transform Method (AHRDTM), which efficiently solves Nonlinear Schrödinger Equations (NLSEs) and reduces computational workload. Therefore, we propose technique that generates a fast-convergent sequence of analytical approximations over a wide time frame. Simultaneously, the number of computed terms has greatly decreased by simplify the handling of nonlinear terms using Adomian polynomials

2. Development of Multistep Modified Reduced Differential Transform Method

Generally, lowercase letters represent the original function. For instance, the function $w(x,t)$, the letter w. In contrast, the capital letter W in the function $W_k(x)$, W, signifies the transformed functions. Essentially, the differential transformation of the function $\mathrm{w(x,t)} = \mathrm{f(x)g(t)}$ is obtained as in Keskin and Oturanç [20],

$$w(x,t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} W_k\,(x)t^k,$$

where the function of $w(x,t)$ is denoted by the symbol $W_k(x)$. The following is a list of definitions that define some of the most fundamental properties of RDTM:

Definition 1. Considering an analytically and continuously differentiable function $w(x,t)$ of time t and space variable x, the differential transformation of $w(x,t)$ is defined as follows

$$W_k(x) = \left[\frac{\partial^k}{\partial t^k} w(x,t)\right]_{t=0} \tag{2}$$

where the transformed function is $W_k(x)$.

Definition 2. The inverse transform of $W_k(x)$ is demonstrated as,

$$w(x,t) = \sum_{k=0}^{\infty} W_k(x)t^k. \tag{3}$$

Combination of Eq. (2) and Eq. (3) leads to the following equation:

$$w(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} w(x,t)\right]_{t=0} t^k. \tag{4}$$

Then, application of the fundamental properties of the MMRDTM to Eq. (1) yields:

$$W_{k+2,i}(x) = \left(\frac{1}{(k+2)(k+1)}\right)\left(\frac{\partial^2}{\partial x^2}\left(W_{k,i}(x)\right) - \sum_{k=0}^{n} A_{k,i} - a(k+1)W_{k+1,i} + h(x,t)\right) \tag{5}$$

The initial condition is expressed as:

$$W_0(x) = f(x). \tag{6}$$

Ray [11] uses the following notation to refer to the nonlinear term:

$$Nw(x,t) = \sum_{n=0}^{\infty} A_n\left(W_0(x), W_1(x), \dots, W_n(x)\right).$$

The work of Kataria and Vellaisamy [18] demonstrates a proposed method for computing Adomian polynomials.

$A_0 = N\big(W_0(x)\big),$

$A_n(W_0(x), W_1(x), \dots, W_n(x)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} N\big(\sum_{k=0}^{n} W_k(x)e^{ikx}\big)\, e^{-in\lambda}\, d\lambda, \quad n \geq 1.$

This approach avoids time-consuming computations with high derivatives. The values are obtained through iterative calculation, enabled by the combination of Eq. (6) and Eq. (5). In addition, the set of values of the inverse transformation, which are denoted by $\{W_k(x)\}_{k=0}^{n}$, provides the following approximation to the solution:

$w(x,t) = \sum_{k=0}^{K} W_k(x)t^k, \qquad t \in [0,T].$

Then, M subintervals $[t_{m-1}, t_m]$ are created by dividing the interval $[0,T]$ into equal step sizes of $s = \frac{T}{M}$ and nodes $t_m = ms$ for $m = 1,2,\dots,M$. The steps outlined below are utilised to compute MMRDTM. Start by solving the initial value problem for the interval $[0, t_1]$ using the modified RDTM. Then, based on the initial conditions,

$w(x,0) = f_0(x),\ w_1(x,0) = f_1(x),$

the approximate result

$w_1(x,t) = \sum_{k=0}^{K} W_{k,1}(x)t^k, \qquad t \in [0,t_1]$

is achieved. At each subinterval $[t_{m-1}, t_m]$, the initial conditions

$w_m(x, t_{m-1}) = w_{m-1}(x, t_{m-1}),$

$(\partial/\partial t)w_m(x, t_{m-1}) = (\partial/\partial t)w_{m-1}(x, t_{m-1}),$

are used for $m \geq 2$ and the multistep RDTM is employed to solve the initial value problem on the interval $[t_{m-1}, t_m]$, with t_0 being replaced by t_{m-1}. The phase is carried out and repeated a number of $m = 1,2,\dots,M$ times, such as, in order to obtain a sequence of approximate solutions denoted by $w_m(x,t)$ where

$w_m(x,t) = \sum_{k=0}^{K} W_{k,m}(x)(t - t_{m-1})^k, \qquad t \in [t_{m-1}, t_m].$

In conclusion, MMRDTM presents the subsequent solutions:

$$w(x,t) = \begin{cases} w_1(x,t), for\ t \in [0, t_1], \\ w_2(x,t), for\ t \in [t_1, t_2], \\ \vdots \\ w_M(x,t), for\ t \in [t_{M-1}, t_M]. \end{cases} .$$

Improved computing speed has made the new method MMRDTM simple to use for all values of s. However, it is important to note that after the step size $s = T$, the MMRDTM reduces to the modified RDTM.

3. Results and Discussions

Two examples were solved using the MMRDTM to highlight the strengths and precision of this technique for solving NLTEs.

Example 1. The second-order NLTE as stated by Yang *et al.,* [21] has been considered,

$$w_{tt} + w_t = 2w_{xx} + w^3 - 2w \tag{7}$$

with the initial condition

$$w(x,0) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\coth\,(x+5),$$

$$w_t(x,0) = \frac{\sqrt{2}}{2}\left(\frac{3}{2} - \frac{3}{2}\coth\big((x+5)\big)^2\right).$$

This exact solution of this equation is $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\coth\left(x + \frac{3t}{2} + 5\right)$.

The application of the fundamental properties of the MMRDTM to Eq. (7) allows us to obtain:

$$W_{k+2,i}(x) = \left(\frac{1}{(k+2)(k+1)}\right)\left(2\frac{\partial^2}{\partial x^2}\left(W_{k,i}(x)\right) + \sum_{k=0}^{n} A_{k,i} - (k+1)W_{k+1,i} - 2W_{k,i}(x)\right). \tag{8}$$

The initial condition is expressed as:

$$W_0(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\coth(x+5) \tag{9}$$

Then, M subintervals $[0, t_1]$ are created by dividing the interval $[0,2]$ into equal step sizes $h = 0.1$, and nodes $t_m = ms$ for $m = 1,2, \ldots, 20$. The steps outlined below are utilised to compute MMRDTM. Start by solving the initial value problem for the interval $[0, t_1]$ using the modified RDTM. Then, based on the initial conditions,

$$w(x,0) = f_0(x),\ w_1(x,0) = f_1(x),$$

the approximate result

$$w_1(x,t) = \sum_{k=0}^{K} W_{k,1}(x)t^k, \qquad t \in [0, t_1]$$

is obtained. At each subinterval $[t_{m-1}, t_m]$, the initial conditions

$$w_m(x, t_{m-1}) = w_{m-1}(x, t_{m-1}),$$

$$(\partial/\partial t)w_m(x, t_{m-1}) = (\partial/\partial t)w_{m-1}(x, t_{m-1}),$$

are used for $m \geq 2$ and the multistep RDTM is employed to solve the initial value problem on the interval $[t_{m-1}, t_m]$, with t_0 being replaced by t_{m-1}. The subsequent multistep scheme for iterative application $w(x,0)=f_0(x), w_1(x,0)= a$. The phase is carried out and repeated for $m=1,2,\ldots,20$ times, such as, in order to obtain a sequence of approximate solutions denoted by $w_m(x,t)$ where

$$w_m(x,t)=\sum_{k=0}^{K} W_{k,m}(x)(t-t_{m-1})^k, \qquad t\in[t_{m-1},t_m].$$

In conclusion, MMRDTM presents the subsequent solutions:

$$w(x,t)=\begin{cases} w_1(x,t), for\ t\in[0,t_1] \\ w_2(x,t), for\ t\in[t_1,t_2] \\ \vdots \\ w_M(x,t), for\ t\in[t_{M-1},t_M]. \end{cases}.$$

It is evident that the multistep approximate solutions exhibit superior accuracy compared to MRDTM when juxtaposed with the exact solutions for the nonlinear telegraph equations as evidenced by the performance error analyses achieved by MMRDTM are presented in Table 1.

Table 1
Error comparison of MMRDTM and MRDTM for Example 1 when $x=0.5$

t	Exact Solutions	Absolute Error (MMRDTM)	Absolute Error (MRDTM)
0.1	1.414231060	0	6.2791×10^{-7}
0.2	1.414226525	1.0000×10^{-9}	2.2839×10^{-6}
0.3	1.414223165	2.0000×10^{-9}	4.8069×10^{-6}
0.4	1.414220675	1.3000×10^{-8}	8.2647×10^{-6}
0.5	1.414218831	7.1000×10^{-8}	1.2989×10^{-5}
0.6	1.414217467	2.3400×10^{-7}	1.9703×10^{-5}
0.7	1.414216454	6.6800×10^{-7}	2.9719×10^{-5}
0.8	1.414215705	1.6470×10^{-6}	4.5216×10^{-5}
0.9	1.414215149	3.6430×10^{-6}	6.9613×10^{-5}
1.0	1.414214739	7.3970×10^{-6}	1.0801×10^{-4}
1.1	1.414214433	1.4016×10^{-5}	1.6771×10^{-4}
1.2	1.414214207	2.5066×10^{-5}	2.5887×10^{-4}
1.3	1.414214040	4.2727×10^{-5}	3.9516×10^{-4}
1.4	1.414213916	6.9909×10^{-5}	5.9457×10^{-4}
1.5	1.414213825	1.1043×10^{-4}	8.8026×10^{-4}
1.6	1.414213757	1.6917×10^{-4}	1.2816×10^{-3}
1.7	1.414213706	2.5230×10^{-4}	1.8349×10^{-3}
1.8	1.414213669	3.6753×10^{-4}	2.5853×10^{-3}
1.9	1.414213641	4.9688×10^{-4}	3.5869×10^{-3}
2.0	1.414213621	7.3238×10^{-4}	4.9048×10^{-3}

Figure 1(a) demonstrates the graphical representation of the exact solution. Figure 1(b) and 1(c) illustrate the graphical representations of the approximate solutions derived from the MMRDTM for $t\in[-2,2]$ and $x\in[-2,2]$and MRDTM for $t\in[-2,2]$ and $x\in[-2,2]$ methods, respectively. Based on the results, the proposed method has better approximations than MRDTM in term of accuracy for this type of equation.

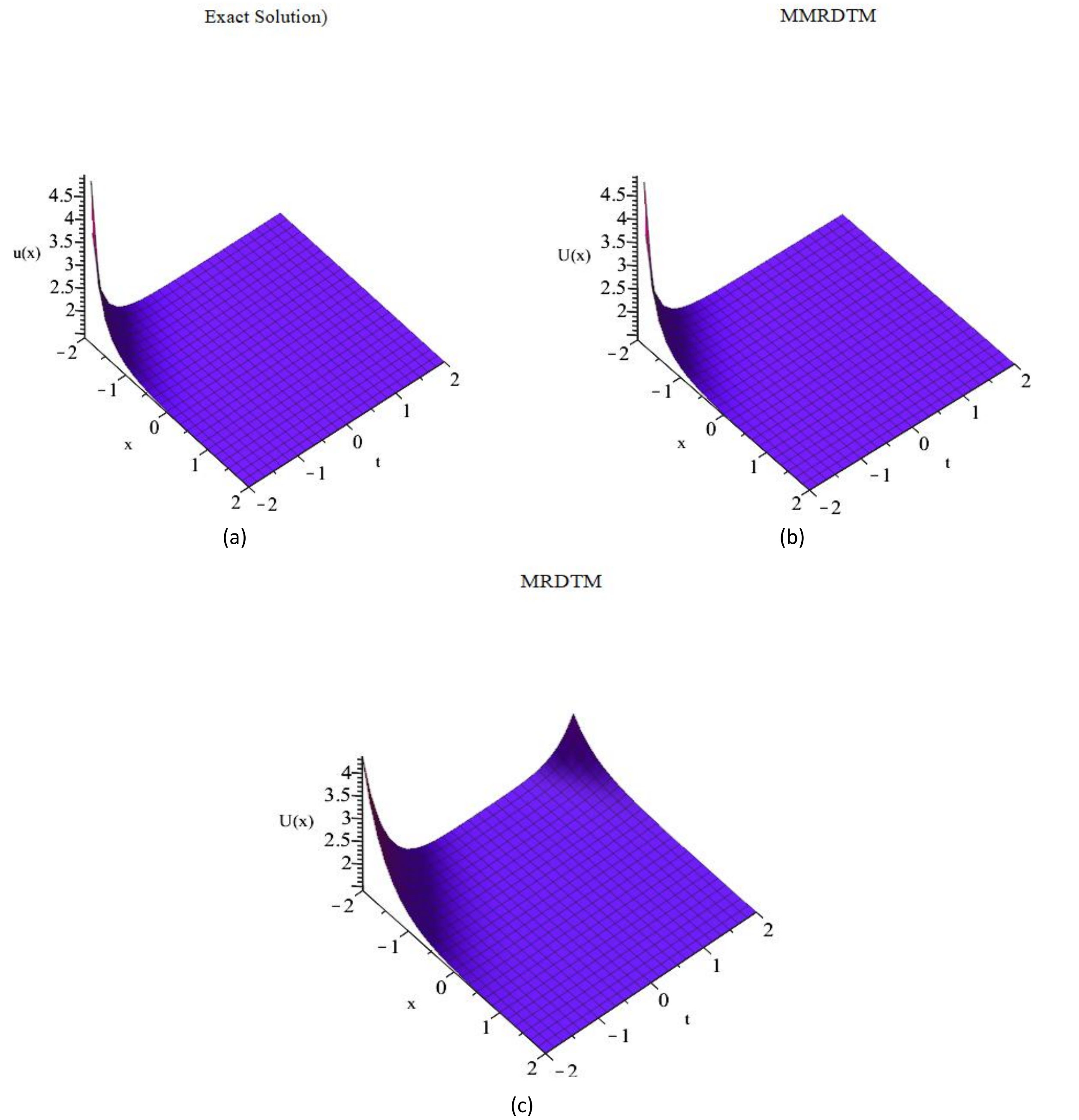


Fig. 1. Comparison graphs of semi-analytical methods with exact solution

Example 2. The second-order NLTE as stated by Al-Badrani *et al.,* [1] is taken into consideration

$$w_{tt} + 2w_t = w_{xx} + w^3 - w \tag{10}$$

subject to the initial condition

$$w(x,0) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{8} + 5\right),$$

$$w_t(x,0) = \frac{3}{16} - \frac{3}{16}\tanh\left(\frac{x}{8} + 5\right)^2.$$

The exact solution of this equation is $\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{x}{8}+\frac{3t}{8}+5\right)$.

The application of the fundamental properties of the MMRDTM to Eq. (10) allows us to obtain:

$$W_{k+2,i}(x)=\left(\frac{1}{(k+2)(k+1)}\right)\left(\frac{\partial^2}{\partial x^2}\left(W_{k,i}(x)\right)+\sum_{k=0}^{n}A_{k,i}-2(k+1)W_{k+1,i}-W_{k,i}(x)\right). \quad (11)$$

From the initial condition, we write

$$W_0(x)=\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{x}{8}+5\right) \quad (12)$$

The following solution is provided by the MMRDTM:

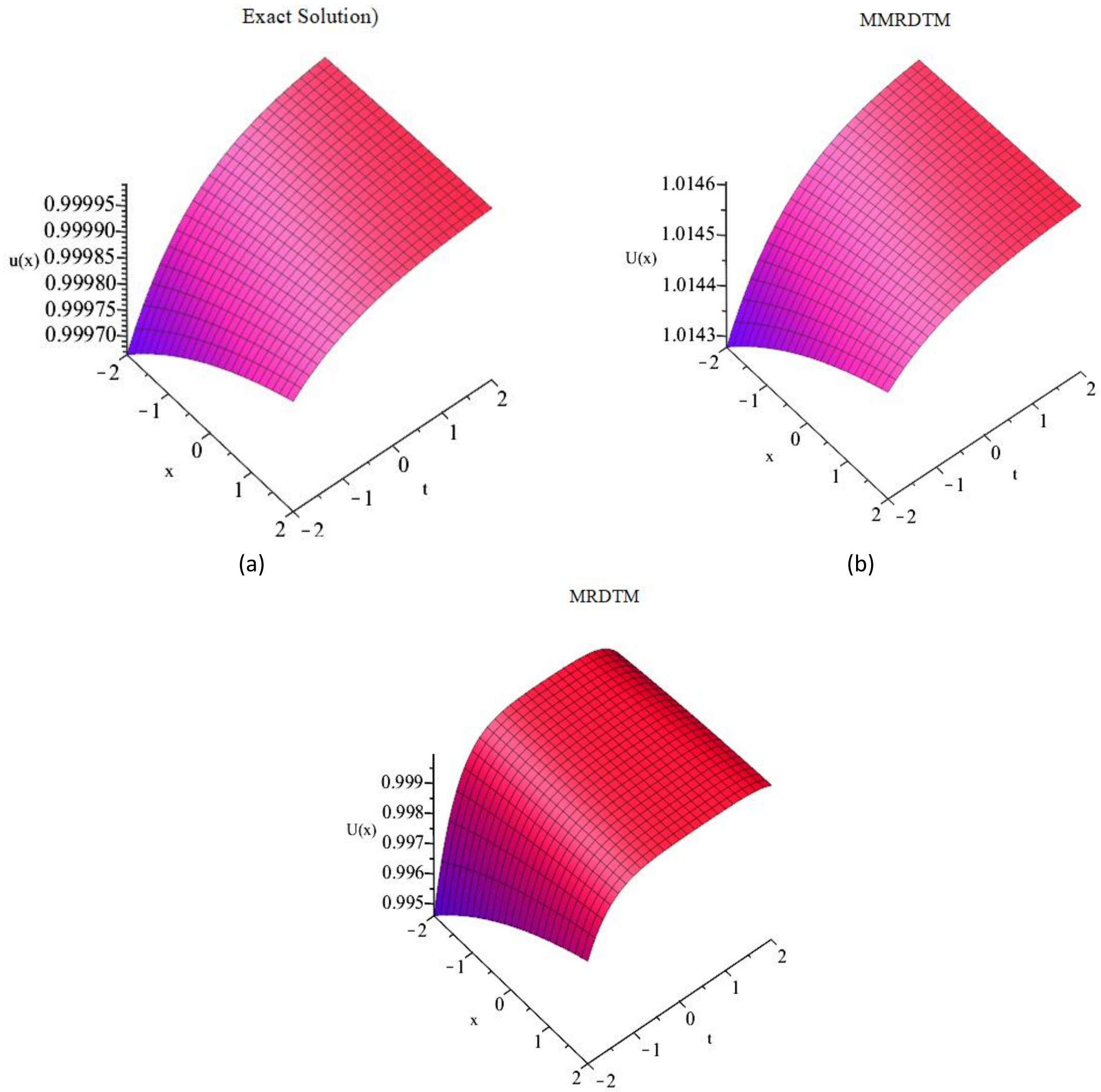
$$w(x,t)=\begin{cases} w_1(x,t), & t\in[0,0.1] \\ w_2(x,t), & t\in[0.1,0.2] \\ \vdots & \vdots \\ w_{20}(x,t), & t\in[1.9,2.0]. \end{cases}.$$

The multistep approximate solutions for this nonlinear telegraph problem closely approximate the exact solutions, as evidenced by the performance error analyses of MMRDTM presented in Table 2.

Table 2
Error comparison of MMRDTM and MRDTM for Example 1 when $x=0.5$

t	Exact Solutions	Absolute Error (MMRDTM)	Absolute Error (MRDTM)
0.1	0.9999628310	3.00000×10^{-10}	5.49600×10^{-7}
0.2	0.9999655166	1.00000×10^{-10}	2.02160×10^{-6}
0.3	0.9999680082	0	4.20370×10^{-6}
0.4	0.9999703198	1.00000×10^{-10}	6.94440×10^{-6}
0.5	0.9999724643	0	1.01424×10^{-5}
0.6	0.9999744539	1.00000×10^{-10}	1.37492×10^{-5}
0.7	0.9999762997	1.000000×10^{-10}	1.77724×10^{-5}
0.8	0.9999780122	3.000000×10^{-10}	2.22915×10^{-5}
0.9	0.9999815421	5.000000×10^{-10}	2.48634×10^{-5}
1.0	0.9999810748	1.00000×10^{-9}	3.36270×10^{-5}
1.1	0.9999824423	2.30000×10^{-9}	4.11888×10^{-5}
1.2	0.9999837110	2.20000×10^{-9}	5.08169×10^{-5}
1.3	0.9999848879	4.70000×10^{-9}	6.34169×10^{-5}
1.4	0.9999859798	9.70000×10^{-9}	8.02057×10^{-5}
1.5	0.9999869928	1.58000×10^{-8}	1.027778×10^{-4}
1.6	0.9999879327	2.56000×10^{-8}	1.331750×10^{-4}
1.7	0.9999888046	3.71000×10^{-8}	1.739749×10^{-4}
1.8	0.9999896136	6.03000×10^{-8}	2.283753×10^{-4}
1.9	0.9999903640	8.40000×10^{-8}	3.002801×10^{-4}
2.0	0.9999910603	9.67000×10^{-8}	3.944209×10^{-4}

Consequently, the exact solution is presented in Figure 2(a). Figure 2(b) and 2(c) illustrate the approximate solutions obtained using MMRDTM for $t\in[-2,2]$ and $x\in[-2,2]$ and MRDTM for $t\in[-2,2]$ and $x\in[-2,2]$ respectively. Based on the results, the proposed method has better approximations than MRDTM in term of accuracy for this type of equation.



(a) (b)

(c)

Fig. 2. Comparison graphs of semi-analytical methods with the exact solution

4. Conclusions

In this paper, MMRDTM was successfully employed to derive a sequence of solutions for second-order nonlinear telegraph equations. The obtained solutions were compared to exact solutions and MRDTM solutions. Moreover, the modification was carried out in a multi-step approach by the replacement of the nonlinear term with its Adomian polynomials. Consequently, the findings and graphic representations showed that the approximations to nonlinear telegraph equations had been obtained with a high degree of accuracy. Hence, it can be asserted that the analytic approximation solutions generated by MMRDTM for this class of equations exhibit enhanced effectiveness, consistency, and accuracy relative to those obtained through MRDTM. The computations presented in this study were executed using the Maple software package.

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