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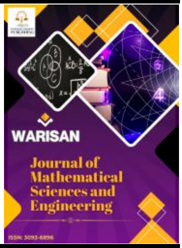

Implementing 3D Modeling: Interpolated B-Spline Surfaces Enhanced with Interval Type-2 Neutrosophic Set Theory

Siti Nur Idara Rosli[1], Mohammad Izat Emir Zulkifly[1,*], Broumi Said[2,3]

[1] Department of Mathematics, Faculty of Sciences, University of Technology Malaysia (UTM), 81310, Johor Bahru, Malaysia
[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco
[3] STIE team Regional Center for the Professions of Education and Training, Casablanca- Settat, Morocco

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ABSTRACT

The interval type-2 neutrosophic set (IT2NS) is a superset of the type-1 neutrosophic set, interval type-2 fuzzy set, and intuitionistic fuzzy set. This paper will demonstrate how to use the interpolation strategy to depict the interval type-2 neutrosophic B-spline surface (IT2NB-sS) model. However, the presence of truth, indeterminacy, and falsity membership functions in neutrosophic features makes the model difficult to visualize. Aside from that, IT2NS properties have upper and lower bounds that make them challenging. This study will use the IT2NS theory to develop the model by introducing an interval type-2 neutrosophic control net relation (IT2NCNR). The IT2NB-sS models will be represented by combining the IT2NCNR with the B-spline basis function. The IT2NCNR's truth, indeterminacy, and falsity memberships are then interpolated for upper and lower bounds to reveal the IT2NB-sSs. The study will conclude by reviewing the algorithm used to construct the IT2NB-sS interpolation models. As a result, the study's findings will result in a prediction model that may be used in various data-gathering applications, including medical applications and real-world data such as bathymetry that involve uncertainty problems.

1. Introduction

As an expansion of the traditional idea of a set and a theory to account for fuzziness (degree of truth) as articulated in natural or human language, Zadeh [1] developed the fuzzy set (FS) theory, as stated in [2,3]. Type-1 FS (T1FS) is widely used and has a meaning connected with uncertainty; nevertheless, prior work has shown that T1FS only represents uncertainty to a limited extent and may not be able to manage or mitigate the consequences of uncertainties seen in some real-world applications, as noted in [4]. In response to this problem, Zadeh [5] suggested extending his previous type-1 fuzzy set (T1FS) theory to incorporate the type-2 fuzzy set (T2FS) theory, which has fuzzy membership grades that allow it to handle uncertainties that T1 finds challenging to manage. A few studies have shown that the results of a T2FS could be superior to those of a T1FS carried out by [6–

* *Corresponding author.*
E-mail address: izatemir@utm.my

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11]. Since using a generic T2FS is computationally expensive, most people only implement interval T2FSs. As highlighted in [4], the computations for interval T2FSs are feasible and controllable.

Intuitionistic fuzzy sets (IFSs), which take membership and non-membership grades into consideration, were introduced by Atanassov [12]. Smarandache [13] extended IFSs to neutrosophic sets (NSs) with truth, indeterminacy, and falsity membership functions to capture the indeterminacy membership grade. A single-valued neutrosophic set (SVNS) was proposed by Wang *et al.,* [14] to deal with ambiguous, indeterminate, and incoherent data. Wang *et al.,* [15] developed interval type-2 neutrosophic sets (IT2NSs) to more accurately evaluate improbability and ambiguity as mentioned in [16]. Interval type-2 trapezoidal neutrosophic numbers via operations were proposed by Touqeer *et al.,* [17]. The study states that the definition and application of an interval neutrosophic set are identical to those of an interval type-2 neutrosophic set. Therefore, the advancement of IT2NS is described in Figure 1.

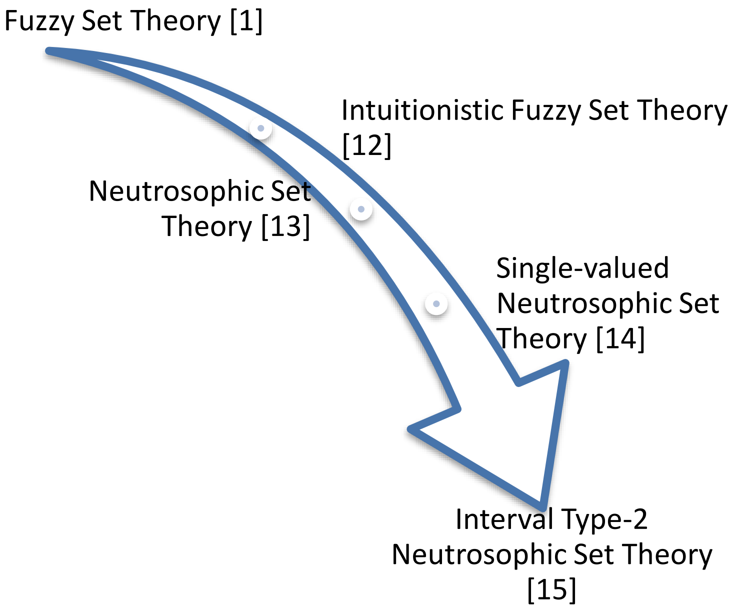


Fig. 1. The advancement of interval type-2 neutrosophic set theory

A B-spline surface is a type of surface representation in computer graphics and geometric modeling, defined by a set of control points and two knot vectors, that maps a unit square to a rectangular surface patch. An essential part of presenting the surfaces is the data set. Before using data collection to construct surface models, any ambiguities must be cleared up. Making geometric models with an IT2NS is a useful method for resolving the problem of making data visible in the face of ambiguity. In geometric modeling with uncertain data, some studies have been done to make sure curves and surfaces are easily useable, as in [18-21]. Fuzzy and intuitionistic fuzzy geometry modeling has been the subject of numerous academic publications, such as [22-28,36,37,41-43]. Aside from that, some works blended the neutrosophic set theory with geometric modeling in [29-33,44,45]. Some academics studied the interval-valued, interval type-2, and deneutrosophication of neutrosophic geometric modeling in [34,46-49]. Meanwhile, this work will use the interpolation method to present an interval type-2 neutrosophic B-spline surface.

Therefore, the research gap is that there are no studies investigating the interpolation of interval type-2 neutrosophic B-spline surface. This study will introduce the models based on the previous study. The primary goal of this study is to develop an interpolation model for an interval type-2 neutrosophic B-spline surface (IT2NB-sS). The IT2NS and its properties must be employed to define the interval type-2 neutrosophic control net relation (IT2NCNR) before constructing the IT2NB-sS. These control nets are utilized to generate models of IT2NB-sSs using the B-spline basis function, which is then depicted through an interpolation technique. The structure of this document is as follows. Some background information about the study was given in Section 1. The basic ideas of IT2NS, IT2NCNR, and interval type-2 data points (IT2DPs) are explained in Section 2. How to interpolate the IT2NB-sS using IT2NCNR is explained in Section 3. A numerical example and a visualization of IT2NB-sS are shown in Section 4. Finally, part 5 will bring this study to a conclusion.

2. Basis Properties

The purpose of this section is to provide the IT2NDPs and IT2NCPs that refer to a dataset. To depict the IT2NBCs, the IT2NDPs are IT2NCPs. Thus, IT2NS, interval type-2 neutrosophic relation (IT2NS), and interval type-2 neutrosophic point (IT2NP) must all be defined before talking about IT2NDPs. The core idea behind IT2NS was first presented by Wang *et al.,* [35]. Later, the formulation of IT2NCPs for neutrosophic points was motivated by Tas and Topal [36,37]. Additionally, Zakaria *et al.,* [38-40] on type-2 fuzzy data points in geometric modeling and Touqeer *et al.,* [17] interval type-2 trapezoidal neutrosophic numbers are the sources of the idea of IT2NDPs.

Definition 1 [35]
Let X be the universal set that elements in X denoted as x. An interval type-2 neutrosophic set (IT2NS) A is expressed by the truth membership function T_A, indeterminacy membership function I_A, and false membership function F_A. Where $x \in X, T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

When X is continuous, an IT2NS A can be expressed as

$$A = \int_X \langle T(x), I(x), F(x) \rangle / x, x \in X \tag{1}$$

When X is discrete, an IT2NS A can be expressed as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i) \rangle / x_i, x_i \in X \tag{2}$$

Definition 2 [35]
Suppose X and Y be a non-empty crisp set. $R(X,Y)$ denoted as interval type-2 neutrosophic relation (IT2NR) in a subset of product space $X \times Y$ and containing the truth membership function $T_R(x,y)$, indeterminacy membership function $I_R(x,y)$ and false membership function $F_R(x,y)$ where $x \in X$ and $y \in Y$, and $T_R(x,y), I_R(x,y), F_R(x,y) \subseteq [0,1]$.

Definition 3 [34]
Suppose A in the space of $x \in X$ is an interval type-2 neutrosophic point (IT2NP) and $x = \{x_i\}$ is a set of IT2NPs where there exists $T_A(x) = [\sup(T_A), \inf(T_A)] : X \to [0,1]$ defining as the supremum and infimum of truth membership, $I_A(x) = [\sup(I_A), \inf(I_A)] : X \to [0,1]$ defining as the supremum

and infimum of indeterminacy membership and $F_A(x) = [\sup(F_A), \inf(F_A)] : X \to [0,1]$ defining as the supremum and infimum of false membership where

$$T_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ a \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \quad (3)$$

$$I_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ b \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \quad (4)$$

$$F_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ c \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \quad (5)$$

Definition 4 [34]
Let $A = \{x | x \text{ interval type-2 neutrosophic point}\}$ and $D = \{D_i | D_i \text{ data point}\}$ is a set of interval type-2 neutrosophic data points with $D_i \in D \subset X$, where X is a universal set and $T_A(D_i) = [\sup(T_A), \inf(T_A)] : D \to [0,1]$ for the truth membership function, which defined as $T_A(D_i) = 1$, $I_A(D_i) = [\sup(I_A), \inf(I_A)] : D \to [0,1]$ for indeterminacy membership function defined as $I_A(D_i) = 1$, $F_A(D_i) = [\sup(F_A), \inf(F_A)] : D \to [0,1]$ for falsity membership function defined as $F_A(D_i) = 1$ and formulated by $D = \{(D_i, T_A(D_i), I_A(D_i) F_A(D_i) | D_i \in \square)\}$. Thus,

$$T_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ a \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \quad (6)$$

$$I_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ b \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \quad (7)$$

$$F_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ c \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \quad (8)$$

For all i and the three memberships, $D_i = \langle D_i^L, D_i, D_i^R \rangle$ with $D_i^L = \langle D_i^{LL}, D_i^L, D_i^{LR} \rangle$ where D_i^{LL}, D_i^L and D_i^{LR} are left-left, left, and left-right of IT2NDP and $D_i^R = \langle D_i^{RL}, D_i^R, D_i^{RR} \rangle$ where D_i^{RL}, D_i^R, and D_i^{RR} are right-left, right, and right-right of IT2NDP respectively. Figure 2 illustrates this with the green triangle representing truth, the blue triangle representing indeterminacy, and the red triangle representing falsehood memberships. The dashed triangle for each membership at the values $[D^L, D, D^R]$

represented the type-1 neutrosophic set for each membership. The upper and lower boundaries for each membership are given by values $\left[D^{LL}, D, D^{RR}\right]$ and $\left[D^{LR}, D, D^{RL}\right]$, respectively.

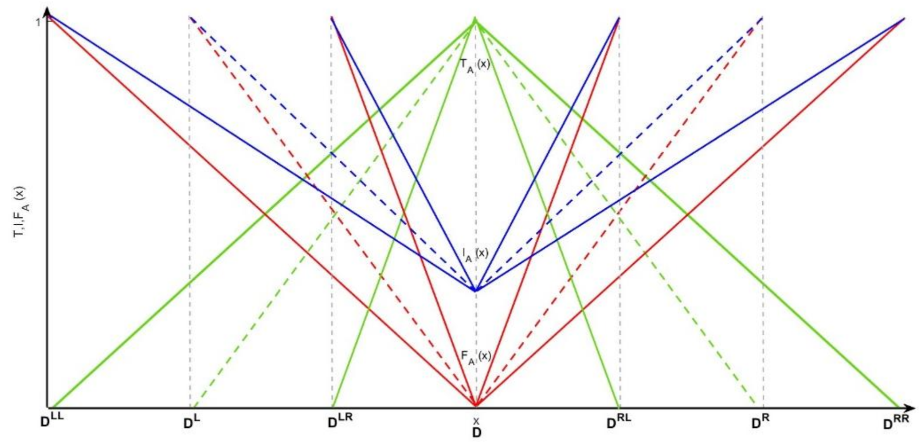


Fig. 2. Interval type-2 neutrosophic data points for truth, indeterminacy, and membership degrees

2.1 Interval Type-2 Neutrosophic Control Net Relation (IT2NCPR)

The interval type-2 neutrosophic control point relation (IT2NCPR) is defined in this section first by using the notion of an interval type-2 neutrosophic set from the research published by Wahab *et al.,* [24,25] in the following way:

Definition 5 [34]
Let $\hat{R}$ be an IT2NPR, then IT2NCPR is defined as a set of point $n+1$ that indicates the positions and coordinates of a location is used to describe the curve and is denoted by

$$\begin{aligned} \hat{P}_i^T &= \left\{\hat{p}_0^T, \hat{p}_1^T, \ldots, \hat{p}_n^T\right\} \\ \hat{P}_i^I &= \left\{\hat{p}_0^I, \hat{p}_1^I, \ldots, \hat{p}_n^I\right\} \\ \hat{P}_i^F &= \left\{\hat{p}_0^F, \hat{p}_1^F, \ldots, \hat{p}_n^F\right\} \end{aligned} \tag{9}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are interval type-2 neutrosophic control points for membership truth, indeterminacy and i is one less than n. Thus, the IT2NCNR can be defined as follows.

Definition 6
Let $\hat{P}$ be an IT2NCPR, and then define an IT2NCNR as points n and m for $\hat{P}$ in their direction, and it can be denoted by $\hat{P}_{i,j}$ that represents the locations of points used to describe the surface and may be written as

$$\hat{P}_{i,j}^{T,I,F} = \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m} \end{bmatrix} \tag{10}$$

3. Interpolation of an Interval Type-2 Neutrosophic B-Spline Surface (IT2NB-sS)

The IT2NCNR and Definition 1 are used to build the IT2NB-sS, which is then used to incorporate the B-spline blending function into a geometric model. The IT2NB-sS stands for an interpolation approach that can be represented mathematically as follows:

Definition 7

Let $\hat{P}_{i,j}^{T,I,F} = \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m} \end{bmatrix}$

where $i = 0,1,...,n$ and $j = 0,1,...,m$ is IT2NCNR. Cartesian Bézier surface is given by:

$$\begin{aligned} BsS^{T}(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}_{i,j}{}^{T} N_i^k(u) M_j^l(w) \\ BsS^{I}(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}_{i,j}{}^{I} N_i^k(u) M_j^l(w) \\ BsS^{F}(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}_{i,j}{}^{F} N_i^k(u) M_j^l(w) \end{aligned} \tag{11}$$

where $N_i^k(u)$ and $M_j^l(w)$ are the Bernstein functions in the u and w parametric directions.

$$\begin{aligned} N_i^1(u) &= \begin{cases} 1 & if \quad \mathrm{u}_i \le u < u_{i+1} \\ 0 & otherwise \end{cases} \\ N_i^k(u) &= \frac{(u-u_i)}{u_{i+k-1}-u_i} N_i^{k-1}(u) + \frac{(u_{i+k}-u)}{u_{i+k}-u_{i+1}} N_{i+1}^{k-1}(u) \\ M_j^1(w) &= \begin{cases} 1 & if \quad w_j \le w < w_{j+1} \\ 0 & otherwise \end{cases} \\ M_j^l(w) &= \frac{(w-w_j)}{w_{j+l-1}-w_i} M_i^{l-1}(w) + \frac{(w_{j+l}-w)}{w_{j+l}-w_{j+1}} M_{j+1}^{l-1}(w) \end{aligned} \tag{12}$$

The surface for the IT2NB-sS will be in the IT2NCNR. As a result, the interpolation procedure is as follows:

$$\begin{bmatrix} \hat{F}_{0,0} & \hat{F}_{0,1} & \cdots & \hat{F}_{0,m} \\ \hat{F}_{1,0} & \hat{F}_{1,1} & \cdots & \hat{F}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{F}_{n,0} & \hat{F}_{n,1} & \cdots & \hat{F}_{n,m} \end{bmatrix} = \begin{bmatrix} BsS(u_0,w_0) & BsS(u_0,w_1) & \cdots & BsS(u_0,w_m) \\ BsS(u_1,w_0) & BsS(u_1,w_1) & \cdots & BsS(u_1,w_m) \\ \vdots & \vdots & \ddots & \vdots \\ BsS(u_n,w_0) & BsS(u_n,w_1) & \cdots & BsS(u_n,w_m) \end{bmatrix} \quad (13)$$

Each $BsS(u_i,w_j)$ can be expressed as a matrix product as follows:

$$BsS(u_i,w_j) = \begin{bmatrix} N_0^n(u_i) & N_1^n(u_i) & \cdots & N_n^n(u_i) \end{bmatrix} \times \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m} \end{bmatrix} \times \begin{bmatrix} M_0^m(w_j) \\ M_1^m(w_j) \\ \vdots \\ M_m^m(w_j) \end{bmatrix} \quad (14)$$

All the separate equations can be combined into a single matrix equation:

$$\hat{F} = M^T \hat{P} N \quad (15)$$

where $\hat{F}$ denotes the given matrix data points from Eq. (13), and $\hat{P}$ denotes the matrix containing the unknown control points $\hat{P}_{i,j}$. The values of the Bernstein polynomials at the given parameters are contained in the matrix M^T and N:

$$M^T = \begin{bmatrix} M_0^n(u_0) & M_1^n(u_0) & \cdots & M_n^n(u_0) \\ M_0^n(u_1) & M_1^n(u_1) & \cdots & M_n^n(u_1) \\ \vdots & \vdots & \ddots & \vdots \\ M_0^n(u_i) & M_1^n(u_i) & \cdots & M_n^n(u_n) \end{bmatrix} \quad (16)$$

$$N = \begin{bmatrix} N_0^m(w_0) & N_0^m(w_0) & \cdots & N_0^m(w_m) \\ N_0^m(w_1) & N_1^m(w_1) & \cdots & N_1^m(w_m) \\ \vdots & \vdots & \ddots & \vdots \\ N_m^m(w_i) & N_m^m(w_i) & \cdots & N_m^m(w_m) \end{bmatrix} \quad (17)$$

Eq. (15) can be easily simplified to

$$\hat{P} = \left(M^T\right)^{-1} \hat{F} \left(N\right)^{-1} \quad (18)$$

4. Visualization of Interval Type-2 Neutrosophic B-Spline Surface (IT2NB-sS)

The IT2NB-sS interpolation model for the truth, indeterminacy, and falsity membership functions is demonstrated in this section. Finally, a method for building the IT2NB-sS will be discussed, and a demonstration of combining all memberships in one axis will be given. Let's look at the matrices below as an example of an IT2NCN dataset. Based on **Definition 4** and Figure 2, $\hat{P}_{3,3}$, $\hat{P}^{L}_{3,3}$, $\hat{P}^{LL}_{3,3}$, $\hat{P}^{LR}_{3,3}$, $\hat{P}^{R}_{3,3}$, $\hat{P}^{RL}_{3,3}$, $\hat{P}^{RL}_{3,3}$ and $\hat{P}^{RR}_{3,3}$ as mean, left, left-left, left-right, right, right-left, and right-right of 4×4 IT2NCNs respectively for B-spline surfaces with the degree of polynomial $n=3$.

$$\hat{P}_{3,3}=\begin{bmatrix} \langle(-17,17);0.4,0.7,0.2\rangle & \langle(-17,7);0.9,0.3,0.1\rangle & \langle(-17,-7);0.4,0.4,0.5\rangle & \langle(-17,-17);0.6,0.5,0.2\rangle \\ \langle(-7,17);0.6,0.4,0.3\rangle & \langle(-7,7);0.8,0.2,0.3\rangle & \langle(-7,-7);0.5,0.5,0.3\rangle & \langle(-7,-17);0.7,0.4,0.2\rangle \\ \langle(7,17);0.6,0.2,0.5\rangle & \langle(7,7);0.8,0.4,0.1\rangle & \langle(7,-7);0.5,0.7,0.1\rangle & \langle(7,-17);0.5,0.3,0.5\rangle \\ \langle(17,17);0.7,0.3,0.3\rangle & \langle(17,7);0.4,0.6,0.3\rangle & \langle(17,-7);0.4,0.6,0.3\rangle & \langle(17,-17);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{L}_{3,3}=\begin{bmatrix} \langle(-19,15);0.4,0.7,0.2\rangle & \langle(-19,5);0.9,0.3,0.1\rangle & \langle(-19,-9);0.4,0.4,0.5\rangle & \langle(-19,-19);0.6,0.5,0.2\rangle \\ \langle(-9,15);0.6,0.4,0.3\rangle & \langle(-9,5);0.8,0.2,0.3\rangle & \langle(-9,-9);0.5,0.5,0.3\rangle & \langle(-9,-19);0.7,0.4,0.2\rangle \\ \langle(5,15);0.6,0.2,0.5\rangle & \langle(5,5);0.8,0.4,0.1\rangle & \langle(5,-9);0.5,0.7,0.1\rangle & \langle(5,-19);0.5,0.3,0.5\rangle \\ \langle(15,15);0.7,0.3,0.3\rangle & \langle(15,5);0.4,0.6,0.3\rangle & \langle(15,-9);0.4,0.6,0.3\rangle & \langle(15,-19);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{LR}_{3,3}=\begin{bmatrix} \langle(-21,13);0.4,0.7,0.2\rangle & \langle(-21,3);0.9,0.3,0.1\rangle & \langle(-21,-11);0.4,0.4,0.5\rangle & \langle(-21,-21);0.6,0.5,0.2\rangle \\ \langle(-11,13);0.6,0.4,0.3\rangle & \langle(-11,3);0.8,0.2,0.3\rangle & \langle(-11,-11);0.5,0.5,0.3\rangle & \langle(-11,-21);0.7,0.4,0.2\rangle \\ \langle(3,13);0.6,0.2,0.5\rangle & \langle(3,3);0.8,0.4,0.1\rangle & \langle(3,-11);0.5,0.7,0.1\rangle & \langle(3,-21);0.5,0.3,0.5\rangle \\ \langle(13,13);0.7,0.3,0.3\rangle & \langle(13,3);0.4,0.6,0.3\rangle & \langle(13,-11);0.4,0.6,0.3\rangle & \langle(13,-21);0.7,0.4,0.2\rangle \end{bmatrix}$$

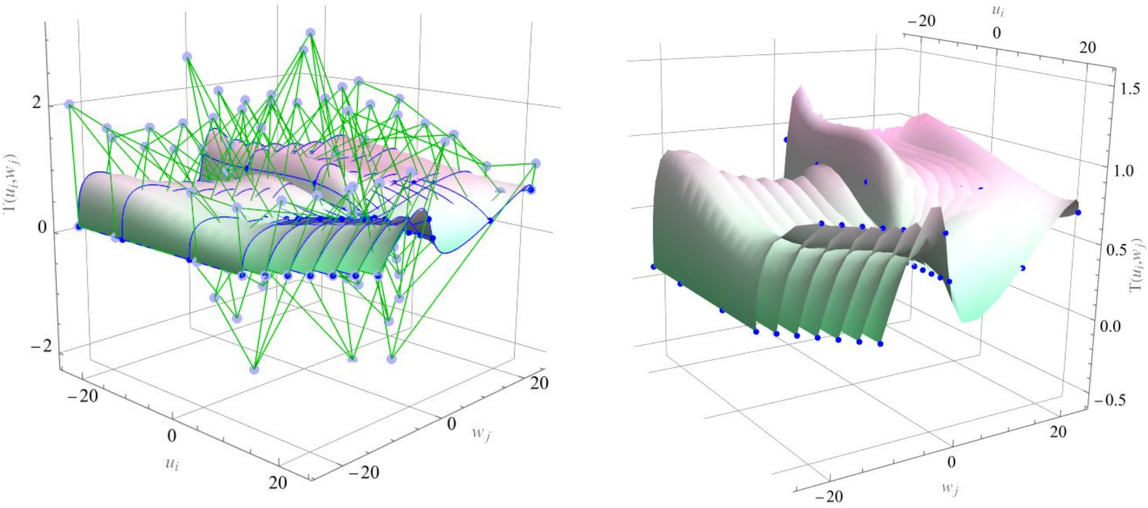
$$\hat{P}^{LL}_{3,3}=\begin{bmatrix} \langle(-23,11);0.4,0.7,0.2\rangle & \langle(-23,1);0.9,0.3,0.1\rangle & \langle(-23,-13);0.4,0.4,0.5\rangle & \langle(-23,-23);0.6,0.5,0.2\rangle \\ \langle(-13,11);0.6,0.4,0.3\rangle & \langle(-13,1);0.8,0.2,0.3\rangle & \langle(-13,-13);0.5,0.5,0.3\rangle & \langle(-13,-23);0.7,0.4,0.2\rangle \\ \langle(1,11);0.6,0.2,0.5\rangle & \langle(1,1);0.8,0.4,0.1\rangle & \langle(1,-13);0.5,0.7,0.1\rangle & \langle(1,-23);0.5,0.3,0.5\rangle \\ \langle(11,11);0.7,0.3,0.3\rangle & \langle(11,1);0.4,0.6,0.3\rangle & \langle(11,-13);0.4,0.6,0.3\rangle & \langle(11,-23);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{R}_{3,3}=\begin{bmatrix} \langle(-15,19);0.4,0.7,0.2\rangle & \langle(-15,9);0.9,0.3,0.1\rangle & \langle(-15,-5);0.4,0.4,0.5\rangle & \langle(-15,-15);0.6,0.5,0.2\rangle \\ \langle(-5,19);0.6,0.4,0.3\rangle & \langle(-5,9);0.8,0.2,0.3\rangle & \langle(-5,-5);0.5,0.5,0.3\rangle & \langle(-5,-15);0.7,0.4,0.2\rangle \\ \langle(9,19);0.6,0.2,0.5\rangle & \langle(9,9);0.8,0.4,0.1\rangle & \langle(9,-5);0.5,0.7,0.1\rangle & \langle(9,-15);0.5,0.3,0.5\rangle \\ \langle(19,19);0.7,0.3,0.3\rangle & \langle(19,9);0.4,0.6,0.3\rangle & \langle(19,-5);0.4,0.6,0.3\rangle & \langle(19,-15);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{RL}_{3,3}=\begin{bmatrix} \langle(-13,21);0.4,0.7,0.2\rangle & \langle(-13,11);0.9,0.3,0.1\rangle & \langle(-13,-3);0.4,0.4,0.5\rangle & \langle(-13,-13);0.6,0.5,0.2\rangle \\ \langle(-3,21);0.6,0.4,0.3\rangle & \langle(-3,11);0.8,0.2,0.3\rangle & \langle(-3,-3);0.5,0.5,0.3\rangle & \langle(-3,-13);0.7,0.4,0.2\rangle \\ \langle(11,21);0.6,0.2,0.5\rangle & \langle(11,11);0.8,0.4,0.1\rangle & \langle(11,-3);0.5,0.7,0.1\rangle & \langle(11,-13);0.5,0.3,0.5\rangle \\ \langle(21,21);0.7,0.3,0.3\rangle & \langle(21,11);0.4,0.6,0.3\rangle & \langle(21,-3);0.4,0.6,0.3\rangle & \langle(21,-13);0.7,0.4,0.2\rangle \end{bmatrix}$$

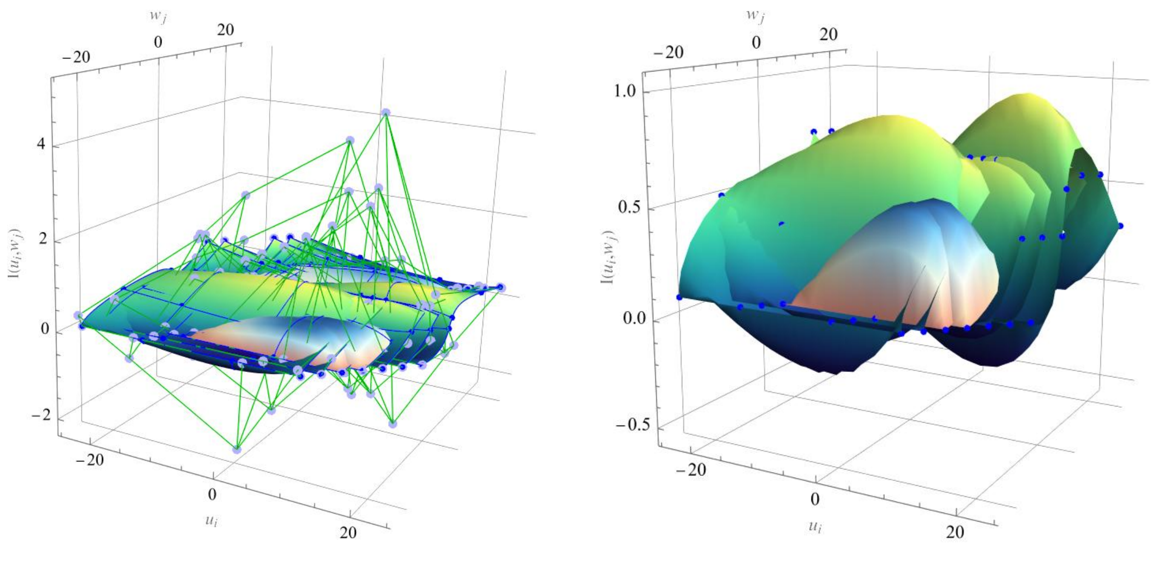
$$\hat{P}^{RR}_{3,3}=\begin{bmatrix} \langle(-11,23);0.4,0.7,0.2\rangle & \langle(-11,13);0.9,0.3,0.1\rangle & \langle(-11,-1);0.4,0.4,0.5\rangle & \langle(-11,-11);0.6,0.5,0.2\rangle \\ \langle(-1,23);0.6,0.4,0.3\rangle & \langle(-1,13);0.8,0.2,0.3\rangle & \langle(-1,-1);0.5,0.5,0.3\rangle & \langle(-1,-11);0.7,0.4,0.2\rangle \\ \langle(13,23);0.6,0.2,0.5\rangle & \langle(13,13);0.8,0.4,0.1\rangle & \langle(13,-1);0.5,0.7,0.1\rangle & \langle(13,-11);0.5,0.3,0.5\rangle \\ \langle(23,23);0.7,0.3,0.3\rangle & \langle(23,13);0.4,0.6,0.3\rangle & \langle(23,-1);0.4,0.6,0.3\rangle & \langle(23,-11);0.7,0.4,0.2\rangle \end{bmatrix}$$

The IT2NB-sSs, which includes membership in truth, indeterminacy, and falsity for both their left and right footprints, is shown in Figure 3 to 5. The MintColors surfaces reflect truth membership, the BlueGreenYellow surfaces indeterminacy, and the RedBlueTones surfaces falsehood. The IT2NB-sSs are shown in Figure 6, together with the corresponding IT2NCNs and memberships. Figure 7 illustrates the operation of an algorithm for producing IT2NBSs.



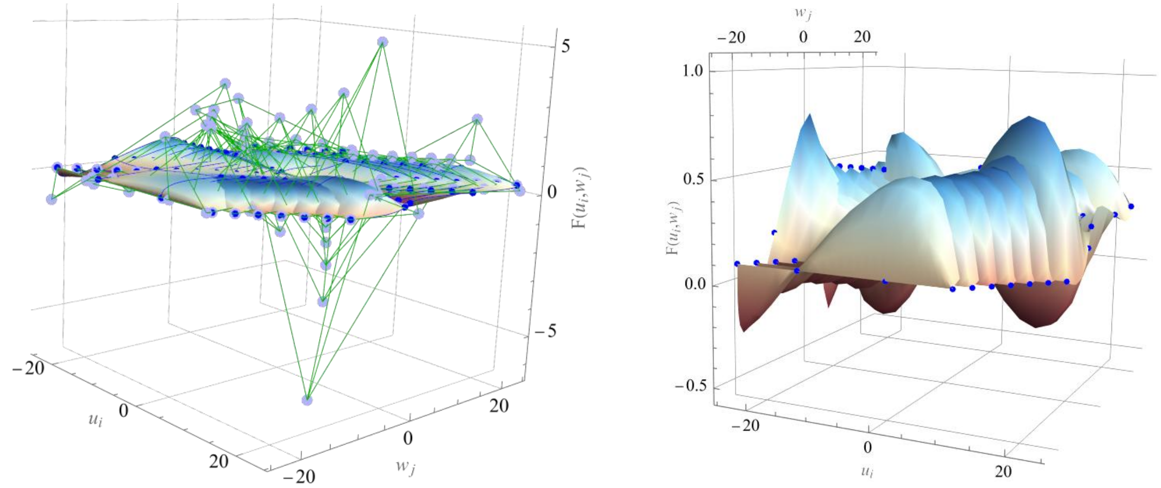
(a) (b)

Fig. 3. (a) IT2NB-sSs for truth membership with its respective IT2NCNs; (b) without IT2NCNs



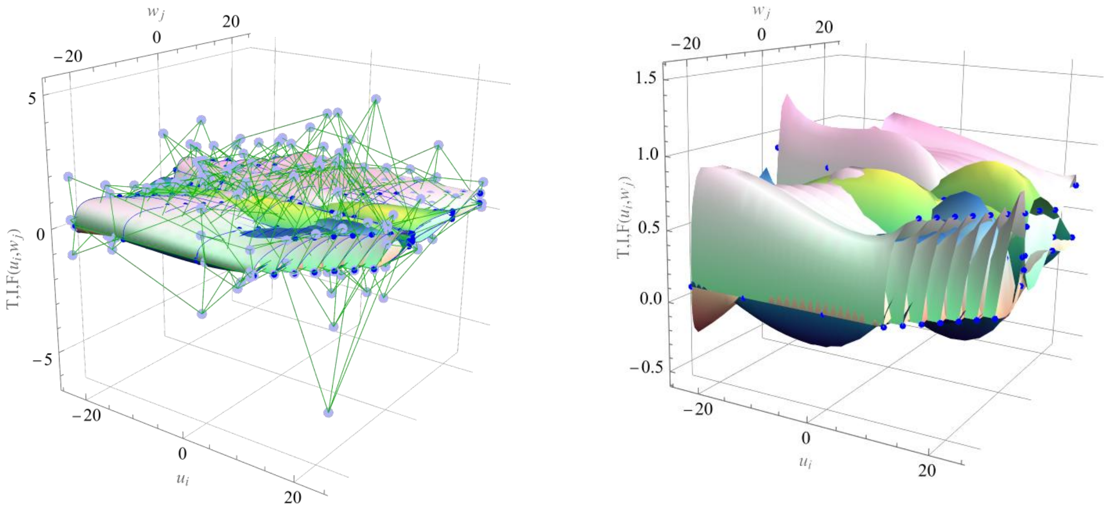
(a) (b)

Fig. 4. IT2NB-sSs for indeterminacy membership with its respective IT2NCNs; (b) without IT2NCNs



(a) (b)

Fig. 5. IT2NB-sSs for falsity membership with its respective IT2NCNs; (b) without IT2NCNs



(a) (b)

Fig. 6. IT2NB-sSs with their respective IT2NCNs and memberships; (b) without IT2NCNs

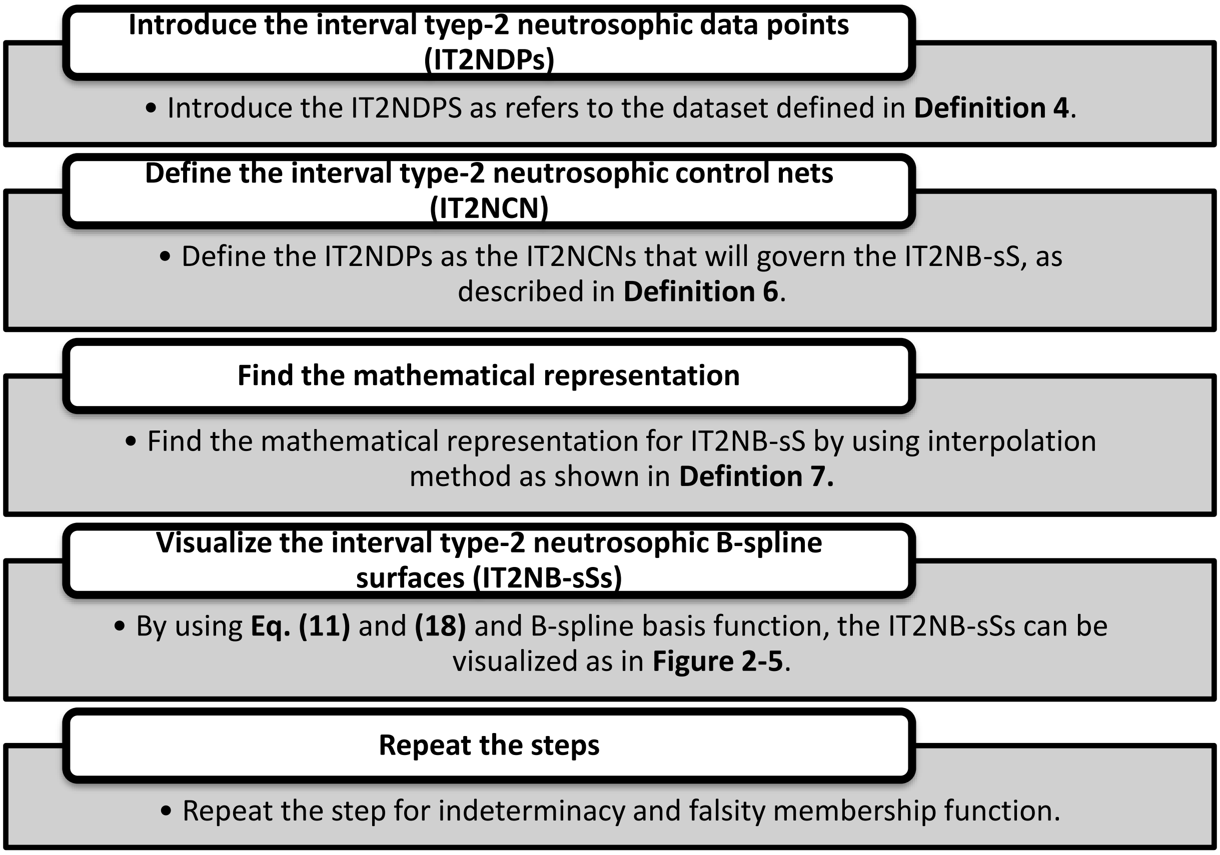


Fig. 7. An algorithm to create the IT2NB-sSs interpolation models

5. Limitation of Study

Despite the promising results of this study, several limitations should be acknowledged. One major challenge is the computational complexity associated with implementing interval type-2 neutrosophic B-spline surfaces (IT2NB-sS). The requirement for high computational resources may restrict the model's feasibility for real-time applications. Additionally, visualization challenges arise due to the complexity of IT2NS properties, which include upper and lower bounds for truth, indeterminacy, and falsity. Effectively representing these features in a clear and interpretable manner remains difficult. Furthermore, the study lacks comparative analysis with alternative surface modeling approaches, such as non-uniform rational B-splines (NURBS) or traditional fuzzy systems, which could help assess the relative advantages and drawbacks of the proposed method. Another limitation is the restricted application scope—while potential applications in medical and bathymetric data modeling are mentioned, practical implementation and validation in real-world scenarios have not been extensively explored. Lastly, the interpolation approach used in this study may introduce approximation errors, which could impact the accuracy and reliability of the generated B-spline surfaces. Addressing these limitations in future research could enhance the robustness and applicability of IT2NB-sS models in various fields.

6. Discussion and Conclusion

This work presented interval type-2 neutrosophic set (IT2NS) principles that are used to construct the interval type-2 neutrosophic control net (IT2NCN), which governs the behaviors of interval type-2 neutrosophic B-spline surfaces (IT2NB-sS). The model introduces an innovative way of displaying uncertain datasets through the lens of IT2NS theory, providing an effective framework for handling complex uncertainty. The ability to model and process uncertain or ambiguous information using IT2NS theory offers a distinct advantage in disciplines where precision is crucial but cannot be guaranteed, making it an essential tool for advancing data analysis and decision-making.

In the context of medical applications, predictive models are critical in diverse areas, ranging from cancer prediction and detection of image blur to disaster warning systems. The integration of IT2NS theory into these areas has the potential to enhance the accuracy and reliability of predictions, enabling more precise decision-making in high-stakes environments. Moreover, the extension to type-2 neutrosophic sets allows for addressing even more complex and uncertain situations that may involve multiple layers of uncertainty.

Future research could explore several routes to refine and extend this work. For example, incorporating more sophisticated geometric models such as non-uniform rational B-splines (NURBS) could enhance the visualization capabilities of the study. These models, which are already well-established in computer graphics and computational geometry, would enable more precise surface modeling and use other methods such as approximation methods. In doing so, they would provide a more precise toolset for visualizing and manipulating complex surfaces in uncertain environments.

By extending the scope of this work to encompass these more advanced methodologies and technologies, future research could contribute significantly to both theoretical advancements and practical applications in areas such as autonomous systems, environmental modeling, robotics, and medical diagnostics, where the need for handling uncertainty is a critical factor in achieving meaningful, actionable results.

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References
[1] Zadeh, Lotfi Asker. "Fuzzy sets." *Information and control* 8, no. 3 (1965): 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X
[2] Pranevicius, Henrikas, Tadas Kraujalis, Germanas Budnikas, and Vytautas Pilkauskas. "Fuzzy rule base generation using discretization of membership functions and neural network." In *Information and Software Technologies: 20th International Conference, ICIST 2014, Druskininkai, Lithuania, October 9-10, 2014. Proceedings 20*, pp. 160-171. Springer International Publishing, 2014. https://doi.org/10.1007/978-3-319-11958-8_13
[3] Chiu, Stephen. "Extracting fuzzy rules from data for function approximation and pattern classification." *Fuzzy information engineering: A guided tour of applications* 9 (1997): 1-10.
[4] Mendel, Jerry M., and Hongwei Wu. "Uncertainty versus choice in rule-based fuzzy logic systems." In *2002 IEEE World Congress on Computational Intelligence. 2002 IEEE International Conference on Fuzzy Systems. FUZZ-IEEE'02. Proceedings (Cat. No. 02CH37291)*, vol. 2, pp. 1336-1341. IEEE, 2002.
[5] Zadeh, Lotfi Asker. "The concept of a linguistic variable and its application to approximate reasoning—I." *Information sciences* 8, no. 3 (1975): 199-249. https://doi.org/10.1016/0020-0255(75)90036-5
[6] Hagras, Hani A. "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots." *IEEE Transactions on Fuzzy Systems* 12, no. 4 (2004): 524-539. https://doi.org/10.1109/TFUZZ.2004.832538
[7] Liang, Qilian, and Jerry M. Mendel. "Interval type-2 fuzzy logic systems: theory and design." *IEEE Transactions on Fuzzy Systems* 8, no. 5 (2000): 535-550. https://doi.org/10.1109/91.873577
[8] Melgarejo, Miguel A., Carlos Andrés Peña-Reyes, and Antonio García. "Computational model and architectural proposal for a hardware type-2 fuzzy system." In *Neural Networks and Computational Intelligence*, pp. 279-284. 2004.

[9] Melin, Patricia, and Oscar Castillo. "A new method for adaptive control of non-linear plants using type-2 fuzzy logic and neural networks." *International Journal of General Systems* 33, no. 2-3 (2004): 289-304. https://doi.org/10.1080/03081070310001633608

[10] Ozen, Turhan, and Jonathan Mark Garibaldi. "Investigating adaptation in type-2 fuzzy logic systems applied to umbilical acid-base assessment." In *Proceedings of 2003 European Symposium on Intelligent Technologies (EUNITE 2003)*, pp. 289-294. 2003.

[11] Wu, Dongrui, and Woei Wan Tan. "A type-2 fuzzy logic controller for the liquid-level process." In *2004 IEEE International Conference on Fuzzy Systems (IEEE Cat. No. 04CH37542)*, vol. 2, pp. 953-958. IEEE, 2004.

[12] Atanassov, Krassimir T., and Krassimir T. Atanassov. *Intuitionistic fuzzy sets*. Physica-Verlag HD, 1999. https://doi.org/10.1007/978-3-7908-1870-3

[13] Smarandache, Florentin. "Neutrosophic set–a generalization of the intuitionistic fuzzy set." *Journal of Defense Resources Management (JoDRM)* 1, no. 1 (2010): 107-116.

[14] Wang, Haibin, Florentin Smarandache, Yanqing Zhang, and Rajshekhar Sunderraman. *Single valued neutrosophic sets*. Infinite study, 2010.

[15] Wang H., Madiraju P., Sunderraman R. and Zhang Y.Q. Interval Neutrosophic Sets. *Department of Computer Science*, State University Atlanta, Georgia, USA. 2004.

[16] Ye, Jun. "Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making." *Journal of intelligent & fuzzy systems* 26, no. 1 (2014): 165-172. https://doi.org/10.3233/IFS-120724

[17] Touqeer, Muhammad, Rimsha Umer, Ali Ahmadian, and Soheil Salahshour. "A novel extension of TOPSIS with interval type-2 trapezoidal neutrosophic numbers using (α, β, γ)-cuts." *RAIRO-Operations Research* 55, no. 5 (2021): 2657-2683. https://doi.org/10.1051/ro/2021133

[18] Yamaguchi, Fujio. *Curves and surfaces in computer-aided geometric design*. Springer Science & Business Media, 2012.

[19] Rogers, David F. *An introduction to NURBS: with historical perspective*. Elsevier, 2000.

[20] Farin, Gerald E. *Curves and surfaces for CAGD: a practical guide*. Morgan Kaufmann, 2002.

[21] Piegl, L., and W. Tiller. "The NURBS Book, Springer-Verlag." *New York* (1995). https://doi.org/10.1007/978-3-642-97385-7

[22] Jacas, Joan, Amadeu Monreal, and Jordi Recasens. "A model for CAGD using fuzzy logic." *International Journal of Approximate Reasoning* 16, no. 3-4 (1997): 289-308. https://doi.org/10.1016/S0888-613X(96)00124-7

[23] Wahab, Abdul Fatah, Jamaludin Md Ali, Ahmad Abdul Majid, and Abu Osman Md Tap. "Fuzzy set in geometric modeling." In *Proceedings. International Conference on Computer Graphics, Imaging and Visualization, 2004. CGIV 2004.*, pp. 227-232. IEEE, 2004.

[24] Wahab, A. F., Ali, J. M. and Majid, A. A. “Fuzzy geometric modeling.” Sixth International Conference on Computer Graphics, *Imaging and Visualization,* 276-280. 2009.

[25] Wahab, Abd Fatah, Jamaludin Md Ali, Ahmad Abd Majid, and Abu Osman Md Tap. "Penyelesaian Masalah Data Ketakpastian Menggunakan Splin-B Kabur." *Sains Malaysiana* 39, no. 4 (2010): 661-670.

[26] Jacas, Joan, Amadeu Monreal, and Jordi Recasens. "A model for CAGD using fuzzy logic." *International Journal of Approximate Reasoning* 16, no. 3-4 (1997): 289-308. https://doi.org/10.1016/S0888-613X(96)00124-7

[27] Bidin, Mohd Syafiq, Abd Fatah Wahab, Mohammad Izat Emir Zulkifly, and Rozaimi Zakaria. "Generalized Fuzzy Linguistic Bicubic B-Spline Surface Model for Uncertain Fuzzy Linguistic Data." *Symmetry* 14, no. 11 (2022): 2267. https://doi.org/10.3390/sym14112267

[28] Zakaria, Rozaimi, Abd Fatah Wahab, Isfarita Ismail, and Mohammad Izat Emir Zulkifly. "Complex uncertainty of surface data modeling via the type-2 fuzzy B-spline model." *Mathematics* 9, no. 9 (2021): 1054. https://doi.org/10.3390/math9091054

[29] Rosli, Siti Nur Idara Binti, and Mohammad Izat Emir Bin Zulkifly. "A Neutrosophic Approach for B-Spline Curve by Using Interpolation Method." *Neutrosophic Systems with Applications* 9 (2023): 29-40. https://doi.org/10.61356/j.nswa.2023.43

[30] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Neutrosophic Bicubic B-spline surface interpolation model for uncertainty data." *Neutrosophic Systems with Applications* 10 (2023): 25-34. https://doi.org/10.61356/j.nswa.2023.69

[31] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "3-Dimensional quartic Bézier curve approximation model by using neutrosophic approach." *Neutrosophic Systems with Applications* 11 (2023): 11-21. https://doi.org/10.61356/j.nswa.2023.78

[32] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Neutrosophic Bicubic Bezier Surface Approximation Model for Uncertainty Data." *Matematika* (2023): 281-291. https://doi.org/10.11113/matematika.v39.n3.1502

[33] Idara Rosli, Siti Nur, and Mohammadi Izati Emir iZulkifly. "Neutrosophic B-spline Surface Approximation Model for 3-Dimensional Data Collection." *Neutrosophic Sets and Systems* 63, no. 1 (2024): 7.

[34] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly Zulkifly. "Interval Neutrosophic Cubic Bézier Curve Approximation Model for Complex Data." *Malaysian Journal of Fundamental and Applied Sciences* 20, no. 2 (2024): 336-346. https://doi.org/10.11113/mjfas.v20n2.3240

[35] Wang, Haibin, Y-Q. Zhang, and Rajshekhar Sunderraman. "Truth-value based interval neutrosophic sets." In *2005 IEEE International Conference on Granular Computing*, vol. 1, pp. 274-277. IEEE, 2005. https://doi.org/10.1109/GRC.2005.1547284

[36] Tas, Ferhat, and Selçuk Topal. "Bezier curve modeling for neutrosophic data problem." *Neutrosophic Sets and Systems* 16 (2017): 3-5. https://doi.org/10.20944/preprints201704.0176.v1

[37] Topal, Selçuk, and Ferhat Tas. "Bezier surface modeling for neutrosophic data problems." *Neutrosophic Sets and Systems* 19 (2018): 19-23.

[38] Zakaria, Rozaimi, Abd Fatah Wahab, and R. U. Gobithaasan. "The Representative curve of type-2 fuzzy data point modeling." *Modern Applied Science* 7, no. 5 (2013): 60-71. https://doi.org/10.5539/mas.v7n5p60

[39] Zakaria, Rozaimi, Abd Wahab, and Rudrusamy U. Gobithaasan. "Normal type-2 fuzzy rational b-spline curve." *arXiv preprint arXiv:1304.7868* (2013). https://doi.org/10.1063/1.4887635

[40] Zakaria, Rozaimi, Abd Wahab, and Rudrusamy U. Gobithaasan. "Perfectly normal type-2 fuzzy interpolation B-spline curve." *arXiv preprint arXiv:1305.0001* (2013). https://doi.org/10.1063/1.4887635

[41] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Fuzzification and Defuzzification of Intuitionistic Fuzzy B-Spline Curve Approximation Model for Visualization of Complex Uncertainty Data." In *2024 5th International Conference on Artificial Intelligence and Data Sciences (AiDAS)*, pp. 498-503. IEEE, 2024.

[42] Sahrom, Nor Ashikin, Mohammad Izat Emir Zulkifly, and Siti Nur Idara Rosli. "Interval-Valued Fuzzy Bézier Surface Approximation." In *2024 5th International Conference on Artificial Intelligence and Data Sciences (AiDAS)*, pp. 1-5. IEEE, 2024.

[43] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly Zulkifly. "Interval Neutrosophic Cubic Bézier Curve Approximation Model for Complex Data." *Malaysian Journal of Fundamental and Applied Sciences* 20, no. 2 (2024): 336-346.

[44] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Neutrosophic Bézier Curve Model for Uncertainty Problem Using Approximation Approach." In *ITM Web of Conferences*, vol. 67, p. 01029. EDP Sciences, 2024. https://doi.org/10.1051/itmconf/20246701029

[45] Rosli, Siti Nuri Idara, and Mohammad Izat Emiri Zulkifly. "Bézier Curve Interpolation Model for Complex Data by Using Neutrosophic Approach." *EDUCATUM Journal of Science, Mathematics and Technology* 12, no. 1 (2025): 1-9. https://doi.org/10.37134/ejsmt.vol12.1.1.2025

[46] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Deneutrosophication of Neutrosophic Bézier Surface Approximation Model." *Neutrosophic Sets and Systems* 72, no. 1 (2024): 18. https://doi.org/10.5281/zenodo.13564500

[47] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Deneutrosophication of neutrosophic B-spline surface approximation model." In *AIP Conference Proceedings*, vol. 3244, no. 1. AIP Publishing, 2024.

[48] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Visualization of 3-dimensional Cubic B-spline Surface Approximation Model by using Interval Type-2 Neutrosophic Set Theory." In *3rd International Conference on Frontiers in Academic Research*, pp. 372-380. 2024.

[49] Rosli, Siti Nur Idara, and Mohammad Izat Emir Zulkifly. "Approximation of Interval Type-2 Neutrosophic Bézier Surface Model for Uncertainty Data." *International Journal of Advanced Research in Computational Thinking and Data Science* 3, no. 1 (2024): 20-31. https://doi.org/10.37934/ctds.3.1.2031