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Fuzzy Interpolation Bezier Curve Model of Shoreline Change Rate

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ABSTRACT

Shoreline dynamics are critical for coastal management. Quantifying shoreline change rates accurately presents a challenge, often hindered by inherent uncertainty in measurement data and natural variability. This research aims to develop a robust model capable of handling such imprecision. A Fuzzy Interpolation Bezier Curve model is proposed and utilized. This method integrates fuzzy numbers to represent uncertain shoreline positions data and employs the interpolation of Bezier curves for a smooth curve between data points. Principal results demonstrate the model's effectiveness in generating fuzzy shoreline change rates, reflecting the inherent uncertainty. The study concludes that this fuzzy Bezier approach offers a valuable tool for analyzing and predicting shoreline changes under conditions of uncertainty, aiding coastal planning.

1. Introduction

Coastal zones represent some of the most dynamic and vulnerable ecosystems on our planet, undergoing continual changes driven by a combination of natural processes—such as erosion, sedimentation, tidal fluctuations, and storm surges—and human activities, including urban development, industrial construction, and resource extraction. It's important to note that these human activities significantly contribute to the vulnerability of these ecosystems from previous studies [1,2]. The shorelines, functioning as the critical interface between terrestrial and marine environments, are particularly susceptible to disturbances from these various forces which have been mentioned by previous studies by the researchers [3-6].

Accurate monitoring and forecasting of shoreline changes are essential for effective coastal zone management, which plays a crucial role in disaster mitigation and promoting sustainable development practices that balance ecological integrity with human needs, as discussed by the researchers [5,7-8]. A comprehensive approach is necessary to protect coastal ecosystems, especially in light of rising sea levels and climate-related impacts, as discussed in a report [9]. Proactive strategies should encompass not only scientific observation but also community engagement and

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policy support to foster resilient coastal communities as mentioned by Mastrorillo *et al.,* [10]. By integrating ecological, social, and economic considerations, we can enhance the sustainability of these vital areas discussed by McGranahan *et al.,* [11].

Traditional approaches for evaluating shoreline change rates, including the End Point Rate (EPR), Linear Regression Rate (LRR), and the Digital Shoreline Analysis System (DSAS), predominantly depend on meticulously collected historical shoreline data to project future trends which were taken by Thieler *et al.,* [12]. Although these methods have gained widespread acceptance and application among coastal researchers and managers, they often fail to adequately address the inherent uncertainties accompanying shoreline data. These uncertainties arise from various sources, including measurement errors, variations over time, and the intricate interplay of natural forces, such as wave action, tidal fluctuations, and sediment movement, alongside human influences like coastal development and erosion mitigation efforts were discussed by Crowell *et al.,* [13]. This oversight in accounting for uncertainties can lead to skewed predictions, potentially jeopardizing the effectiveness of coastal management strategies to preserve vulnerable coastlines and ecosystems.

In response to the complex challenges posed by shoreline changes, this study introduces an innovative methodology that seamlessly integrates fuzzy numbers with Bezier curve interpolation to enhance the modeling of these dynamic environments. Fuzzy numbers which are a fundamental construct derived from the principles of fuzzy sets, offer a sophisticated mathematical framework designed to represent and manipulate uncertain or imprecise data which mentioned by the researches in [14,15]. By weaving fuzzy numbers into the fabric of the modeling process, this novel approach significantly improves the capacity to manage the uncertainties inherent in shoreline data. This, in turn, fosters the development of more robust and reliable predictions, thereby contributing to a clearer understanding of shoreline dynamics and their implications for coastal management.

On the other hand, Bezier curves are parametric curves widely used in computer graphics and geometric modeling to represent smooth transitions between points which were defined by Farin [16]. Their flexibility and mathematical properties make them well-suited for interpolating shoreline data, which often exhibits complex and non-linear patterns. By combining fuzzy numbers with Bezier curve interpolation, the proposed model can capture both the uncertainty and the smoothness of shoreline changes, providing a more accurate representation of shoreline dynamics.

Shoreline change models today often struggle to deal with the uncertainty and imprecision in shoreline data, as well as the complicated shapes and patterns that coastlines naturally have. This is particularly challenging when estimating information between limited measurements taken along the coast. Therefore, this study is significant because it introduces a novel approach that explicitly utilizes the fuzzy numbers concept to define and represent data uncertainty. Meanwhile, the interpolation of Bezier curves is applied to model complex shapes and changes. By combining these two methods, the research aims to create more realistic models of shoreline evolution. This improvement will lead to better predictions and more informed coastal management decisions by effectively accounting for uncertainty during the modeling process.

The primary objective of this study is to develop and validate a fuzzy number-based Bezier curve interpolation model for predicting shoreline change rates. The model is applied to a case study of a coastal region experiencing significant shoreline changes, demonstrating its effectiveness in capturing the complexities of shoreline dynamics. The results are compared with those obtained from traditional methods, highlighting the advantages of the proposed approach.

This paper is organized as follows: Section 2 details the methodology of the process in developing the proposed model. Section 3 as result that generate from Section 2 including the mathematical equation where presents the application of the model to a case study, followed by a discussion of the results in Section 4. Finally, Section 5 concludes the paper with a summary of the findings and recommendations for future research.

2. Methodology

2.1 Data Collection

Data on shoreline conditions were gathered through various methods, including satellite imagery and historical maps such as Google Maps. This study focuses on coastal regions that have exhibited noteworthy erosion and accretion trends over the past twenty years.

2.2 Fuzzy Number Framework

The use of fuzzy numbers were discussed by Zimmermann [17] representation for shoreline positions offers a valuable approach to managing uncertainties. By expressing shoreline positions as triangular and trapezoidal fuzzy numbers, we effectively capture the variability introduced by measurement errors and environmental factors. This method enhances the accuracy of shoreline analysis and improves our understanding of the dynamic nature of coastal environments discussed by researchers [15,18-20].

Fuzzy numbers serve as a useful tool for representing uncertain or imprecise data. A triangular fuzzy number (TFN) is defined by three key parameters: the lower bound (a), the most likely value (d), and the upper bound (c). Understanding these components can enhance our ability to model and analyze data under uncertainty effectively.

Based on the concept of fuzzy numbers, this framework can be extended to define uncertain data, as both uncertain data and fuzzy numbers share the fundamental characteristic of being data. Consequently, this approach enables the precise definition of uncertain data, a process called fuzzy data.

2.3 Bezier Curve Interpolation

Bezier curves are parametric curves widely used in computer graphics, geometric modeling, and interpolation tasks due to their mathematical simplicity and flexibility. They are defined by a set of control points that influence the shape of the curve. Bezier curves are particularly useful for smooth transitions between points, making them ideal for shoreline modeling, animation, and path planning applications.

In the context of modeling shoreline data, employing interpolation methods allows for a shift in focus from traditional control points to actual data points. This approach enhances the accuracy of the curve, ensuring that it directly reflects the underlying data. As a result, the application of Bezier curves in shoreline modeling not only improves the visual representation but also provides a more precise tool for analyzing and understanding shoreline behavior.

3. Result

Section 2 briefly explains how to develop the Fuzzy Interpolation Bezier Curve (FIBC) Model of Shoreline Change Rate and the information and concepts needed. Therefore, in this section, the mathematical formulation will be defined to develop the FIBC model and then used to model the shoreline data. For a better understanding, each definition will be presented visually.

Definition 3.1

Let R be a universal set which R is a real number and A is subset to R. Fuzzy set, $\vec{A}$ in R (number around A in R) called fuzzy number which explained through the alpha-level set (strong and normal alpha-cut) that is if for every alpha, $\alpha \in (0,1]$, there exist set $\vec{A}_\alpha$ in R where $\vec{A}_\alpha = \left\{ x \in R : \mu_{A_\alpha}(x) > \alpha \right\}$ and $\vec{A}_\alpha = \left\{ x \in R : \mu_{A_\alpha}(x) \geq \alpha \right\}$ mentioned by Zakaria *et al.,* [21].

Definition 3.2

If a triangular fuzzy number is represented as $\vec{A} = (a, d, c)$ and $\vec{A}_\alpha$ is an alpha-cut operation of a triangular fuzzy number, then the crisp interval by the alpha-cut operation is obtained as $\vec{A}_\alpha = \left[a^\alpha, b^\alpha \right] = \left[(d-a)\alpha + a, -(c-d)\alpha + c \right]$ with $\alpha \in (0,1]$ where the membership function, $\mu_A(x)$ as discussed by Zakaria *et al.,* [21] can be given as follows.

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{d-a} & \text{for } a \leq x \leq d \\ \dfrac{c-x}{c-d} & \text{for } d \leq x \leq c \\ 0 & \text{for } x > c \end{cases} \tag{1}$$

Figure 1 shows the triangular fuzzy number that illustrates Definition 3.2. It is a commonly used fuzzy number characterized by a triangular membership function.

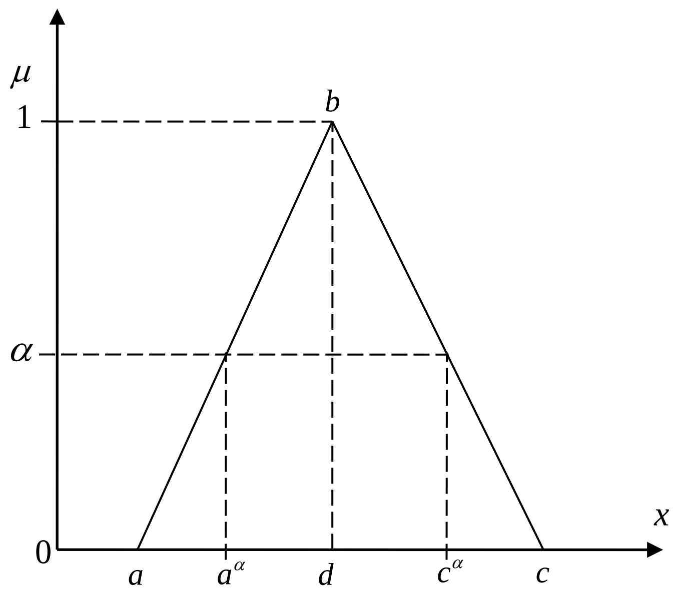

Fig. 1. Triangular fuzzy number, $\vec{A} = (a, d, c)$

Definition 3.3

Let $D=\{(x,y),\ x\in X, y\in Y \mid x,y:\text{fuzzy data}\}$ and $\overleftrightarrow{D}=\{P_i \mid P \text{ is data point}\}$ are the set of fuzzy data points (FDPs) which is $D_i\in D\subset X\times Y\subseteq R$ with R is universal set and $\mu_P(D_i):D\to[0,1]$ is membership function defined as $\mu_P(D_i)=1$ in which $\overleftrightarrow{D}=\{(D_i,\mu_D(D_i))\mid D_i\in R\}$. Therefore,

$$\mu_P(D_i)=\begin{cases} 0 & \text{if} \quad D_i\notin R \\ c\in(0,1) & \text{if} \quad D_i\ \overleftrightarrow{\in}\ R \\ 1 & \text{if} \quad D_i\in R \end{cases} \tag{2}$$

with $\mu_D(D_i)=\langle \mu_P(D_i^{\leftarrow}),\mu_P(D_i),\mu_P(D_i^{\rightarrow})\rangle$ where $\mu_D(D_i^{\leftarrow})$ and $\mu_D(D_i^{\rightarrow})$ are left-grade and right-grade membership values respectively. This can be written as

$$\overleftrightarrow{D}=\{\overleftrightarrow{D}_i=(x_i,y_i)\mid i=0,1,..,n\} \tag{3}$$

for all *i*, $\overleftrightarrow{D}_i=\langle \overleftrightarrow{D}_i^{\leftarrow},D_i,\overleftrightarrow{D}_i^{\rightarrow}\rangle$ with $\overleftrightarrow{D}_i^{\leftarrow}$, D_i and $\overleftrightarrow{D}_i^{\rightarrow}$ are left FDP, crisp data point and right FDP respectively. The procedure in defining FDP is illustrated in Figure 2 as follows.

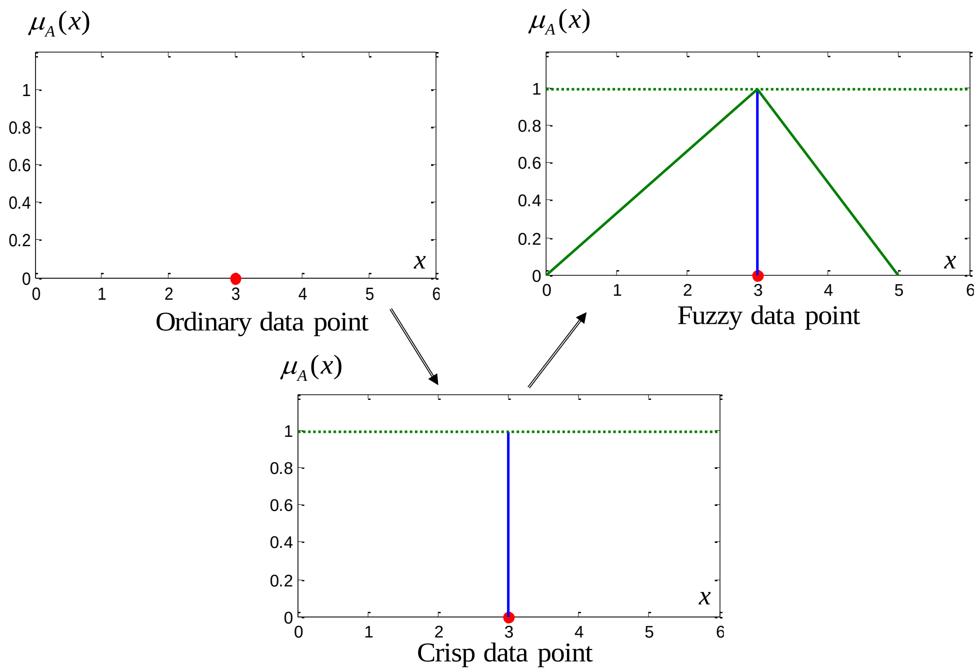

Fig. 2. The process of defining FDPs

Definition 3.4

Let $\overleftrightarrow{D}_i$ be the set of FDPs where $i=0,1,...,m$ and $j=0,1,...,n$. Then, $\overleftrightarrow{D}_{i_\alpha}$ is the alpha-cut operation of fuzzy control point which is given as Eq. (4) where $\alpha\in(0,1]$ with $k=1,2,...,l$.

$$\begin{aligned}\vec{D}_{i_\alpha} &= \left\langle \vec{D}^{\leftarrow}_{i_\alpha}, P_i, \vec{D}^{\rightarrow}_{i_\alpha} \right\rangle \\ &= \left\langle \left[\left(D_i - \vec{D}^{\leftarrow}_i \right)\alpha + \vec{D}^{\leftarrow}_i \right], D_i, \left[-\left(\vec{D}^{\rightarrow}_i - D_i \right)\alpha + \vec{D}^{\rightarrow}_i \right] \right\rangle\end{aligned} \tag{4}$$

Definition 3.5

Let $\vec{D}_\alpha$ is as FDPs after implementing alpha-cut process as defined in Definition 3.4. Then, $\bar{D}_\alpha$ is the defuzzification for all $\vec{D}_\alpha$ that can be define as follows which had been discussed by the researches [22-23].

$$\bar{D}_{i_\alpha} = \frac{\left(\alpha \times \vec{D}^{\leftarrow}_{i_\alpha}\right) + \left(\alpha \times D_i\right) + \left(\alpha \times \vec{D}^{\rightarrow}_{i_\alpha}\right)}{\left(\alpha + \alpha + \alpha\right)} \tag{5}$$

Definition 3.6

Let $\vec{D}_i$ and $\vec{P}_i$ be a set of FDPs and fuzzy control points (FCPs) respectively. Fuzzy interpolation Bezier curve can defined by

$$\vec{B}(t) = \sum_{i=0}^{n} \vec{P}_i B_i^n(t) = \vec{D}_i \tag{6}$$

where $B_i^n(t)$ is the ith Bernstein's polynomial of degree n and $i = 0,1,2,...,m$.

Definition 3.6 explains the defining fuzzy interpolation Bezier curve where the FCPs model needs to find first based on the FDPs given. Normally, the values of FCPs can be generated by implementing the simultaneous equation.

3.2 Fuzzy Interpolation Bezier Curve of Shoreline Change Rate Data

Shoreline change rate (SCR) data is vital for effective coastal management, erosion analysis, and evaluations of climate change impacts. Nevertheless, measuring the shoreline change rate is fraught with uncertainties attributable to various factors. These uncertainties arise from differences in data sources, the methodologies employed for measurement, temporal fluctuations, and natural processes. According to research conducted by [24,25], the shoreline change rate varies between 7 and 50 meters. This variation indicates significant uncertainty, which becomes less definitive when expressed through fuzzy numbers.

Before we proceed to the case study, we need to define Manukan Island, Sabah first. Manukan Island's shoreline is characterized primarily by its long (approx. 1.5km) white sandy beach along the southern coast. The water at the edge is typically clear, turquoise, and calm, with a gentle slope into the sea. Coral reefs and abundant marine life are accessible very close to the shore, particularly near the eastern tip and jetty. The shoreline is developed, featuring jetties and tourist facilities, and is fringed by tropical vegetation. While the southern shore is sandy, other parts of the island's coastline can be rocky. The image of Manukan Island Sabah can be shown as in Figure 3 as follows.

Fig. 3. Manukan Island Sabah close visual (Source: Google Earth Application)

Based on Definition 3.6 and part of data collection of shoreline change rate data at Manukan Island Sabah, the fuzzy interpolation Bezier curve model of shoreline change rate data can be illustrate in Figure 4 as follows.

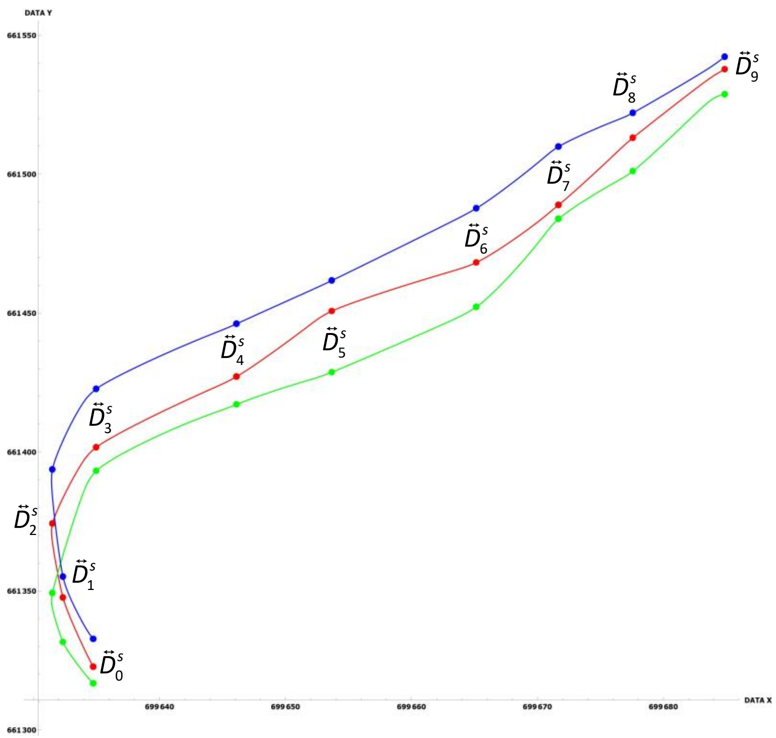

Fig. 4. Fuzzy interpolation Bezier curve of shoreline change rate data

Figure 4 shows a fuzzy interpolation Bezier curve representation of shoreline change rate data in cubic form. The fuzzy data shows that the non-uniform fuzzy interval, which means the fuzzy interval for each data point is different. In this figure, the red curve (crisp data and curve) represents the crisp interpolation of the shoreline change data. The crisp data points are actual recorded values without uncertainty. The red curve represents a smooth interpolated curve passing through these data points using Bezier interpolation. The green and blue curves (as fuzzy boundaries) represent the lower and upper bounds of the fuzzy interpolation. These boundaries account for uncertainties in the shoreline change rate data. The green curve is the lower boundary, showing the minimum possible variation.

The blue curve is the upper boundary, indicating the maximum possible variation. The region between the green and blue curves represents the fuzzy interpolation range.

For the fuzzification process, the alpha-cut method is applied to the shoreline change rate data and then modeled using a fuzzy interpolation Bezier curve. This process can be illustrated by Figure 5.

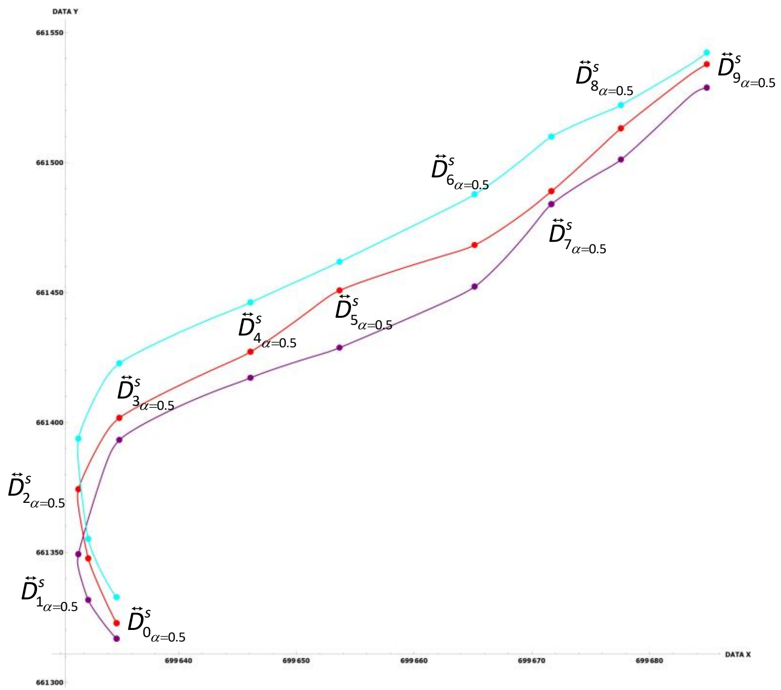

Fig. 5. Fuzzy interpolation Bezier curve of shoreline change rate data

Figure 5 shows the illustration of the alpha-cut (α-cut) process applied to fuzzy interpolation Bezier curves for shoreline change rate data. The cyan curve (upper bound of fuzzy set) represents the upper limit of uncertainty in the shoreline change rate data. It is derived from the highest possible values within a fuzzy range. The purple curve (lower bound of fuzzy set) represents the lower uncertainty limit in the shoreline change rate. It is derived from the lowest possible values within the fuzzy range.

The alpha-cut process extracts a crisp range from the fuzzy shoreline change rate predictions. At $\alpha = 0.5$, we extract a central portion of the fuzzy range, which represents a moderate confidence level. Choosing $\alpha = 0.5$ is particularly important because it represents the Core Median (Central Tendency) where the $\alpha = 0.5$ cut gives the most balanced or likely values in the fuzzy range. It eliminates extreme values while still preserving significant uncertainty. It is also interpreted as a confidence interval with $\alpha = 0.5$ level can be considered as a 50% confidence interval. This means the shoreline change rate is likely within this range, capturing a reasonable degree of uncertainty without extreme values. The other hand, it brings balances between precision and uncertainty which if alpha is too low (e.g., 0.1 or 0.2) than the range is too wide, including many unlikely extreme values and if alpha is too high (e.g., 0.8 or 0.9) than the range is too narrow, excluding some significant uncertainty. But at $\alpha = 0.5$, we balance data accuracy with real-world variability.

The next step in obtaining the crisp fuzzy interpolation Bezier curve of shoreline change rate data is the defuzzification process. The defuzzification process will be applied based on Definition 3.5. The data result of applying the defuzzification process can be illustrated in Figure 6.

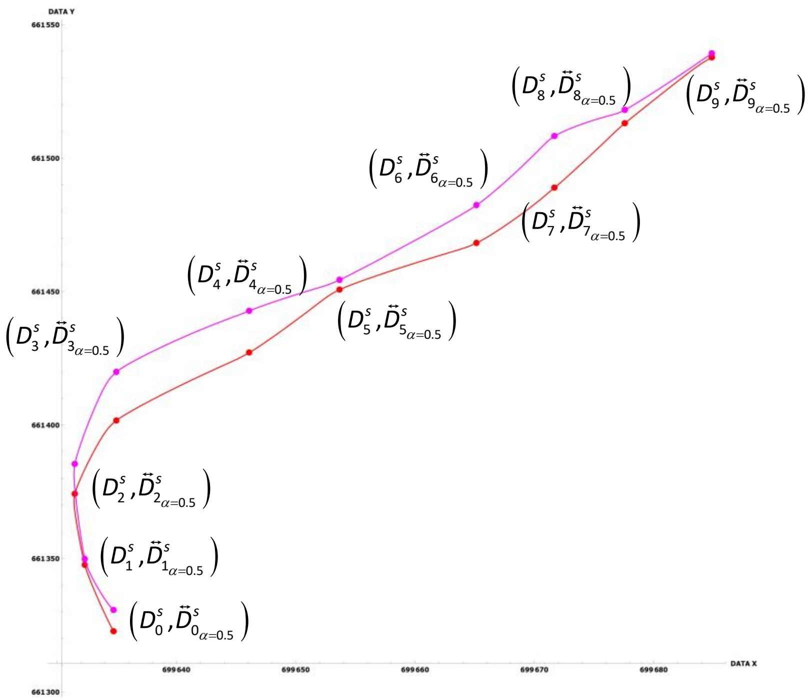

Fig. 6. Fuzzy interpolation Bezier curve of shoreline change rate data

Figure 6 illustrates the defuzzified interpolation Bezier curve of shoreline change rate data, comparing it with the crisp data. The red curve represents the crisp (deterministic) cubic Bezier interpolation of shoreline change rate data, computed directly from measured or modeled shoreline data without considering uncertainty. While this curve provides a precise representation of shoreline changes, it may be overly simplistic, as it strictly follows the original data set without accounting for uncertainties.

In contrast, the purple curve represents the defuzzified interpolation curve, derived from fuzzy interpolation cubic Bezier curve with an alpha-cut value of 0.5. This curve considers the range of uncertainty in the shoreline change rate, offering a more balanced and realistic prediction by averaging the upper and lower fuzzy bounds. It is also slightly adjusted to reflect the influence of uncertainty. The distance between the two curves indicates the level of uncertainty in estimating the shoreline change rate.

To validate the accuracy of the defuzzified fuzzy interpolation cubic Bezier curve compared to the crisp interpolation cubic Bezier curve, we can use the Average Percentage Error (APE) calculation. The APE is used to measure how much the defuzzified interpolation curve data deviates from the crisp interpolation curve data. The formula is:

$$APE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{D_{crisp,i} - D_{defuzzied,i}}{D_{crisp,i}}\right| \times 100 \tag{7}$$

Therefore, the APE for both data and curve give as 0.00141336 for y-axis rather than x-axis that just for the data location. Therefore, the APE value, 0.00141336 (or approximately 0.0014%), represents a very small deviation between the defuzzified fuzzy interpolation cubic Bezier curve and the crisp interpolation cubic Bezier curve. This means that the defuzzified interpolation closely follows the crisp interpolation, with a very low margin of deviation. The defuzzification process (via the alpha-cut at $\alpha = 0.5$) extracts a balanced estimate from the fuzzy model, which, as seen from the APE, retains nearly the same structure as the crisp model.

A low APE value indicates that the defuzzified interpolation cubic Bezier curve effectively preserves the shape and trend of the crisp interpolation cubic Bezier curve despite incorporating fuzzy uncertainty in the modeling process. If the APE were large, it would imply that defuzzification introduced significant deviations from the original shoreline change rate data. However, in this case, the defuzzified model remains highly accurate.

4. Discussion

The proposed model, which integrates fuzzy numbers with cubic Bezier curve interpolation, demonstrates significant advancements in capturing and predicting shoreline changes. One of the key strengths of the model is its ability to successfully capture the uncertainty inherent in shoreline data. Shoreline data is often imprecise due to factors such as measurement errors, tidal fluctuations, and the dynamic nature of coastal processes. Traditional models, which rely on crisp data, struggle to account for these uncertainties, leading to less reliable predictions. By incorporating fuzzy numbers, the proposed model represents shoreline positions as ranges of possible values rather than fixed points. This approach allows the model to handle imprecise data more effectively, providing a more realistic representation of shoreline dynamics. For example, a shoreline position might be represented as a triangular fuzzy number (TFN), where the lower bound, most likely value, and upper bound reflect the range of possible positions. This fuzzy representation ensures that the model accounts for the variability and ambiguity in the data, making it more robust and adaptable to real-world conditions.

Another notable advantage of the model is the use of Bezier curves to provide a smooth representation of shoreline transitions. Shorelines are naturally dynamic and often exhibit complex, non-linear patterns that are difficult to capture using traditional linear or polynomial models. Bezier curves, with their parametric formulation and flexibility, are well-suited for modeling such smooth transitions. By defining a series of control points, the Bezier curve can interpolate between data points, creating a continuous and smooth shoreline path. This smoothness is particularly important for accurately representing natural shoreline features, such as curves, bays, and capes, which are often poorly approximated by traditional methods. The smooth transitions provided by Bezier curves not only improve the visual representation of shorelines but also enhance the accuracy of the model in predicting future shoreline positions.

In summary, the proposed model's integration of fuzzy numbers and Bezier curve interpolation addresses the limitations of traditional shoreline change models by effectively capturing data uncertainty, providing smooth representations of shoreline transitions, and improving prediction accuracy. These advancements make the model a powerful and reliable tool for understanding and managing the dynamic nature of coastal environments.

5. Conclusions

In conclusion, the proposed model, which combines fuzzy numbers and Bezier curve interpolation, offers a robust and innovative approach to shoreline change analysis. By addressing the limitations of traditional methods, the model provides a more accurate, flexible, and reliable framework for understanding and managing the dynamic nature of coastal environments. This research contributes to the growing body of knowledge in coastal geomorphology and provides a valuable tool for addressing the challenges of coastal management in an era of increasing environmental change. Future work will focus on applying the model to larger datasets and integrating additional environmental factors and also use complicated functions such as B-spline and rational Bezier.

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